SIZE STANDARDIZATION BY

PREFERRED NUMBERS

By C. F. Hirschfeld   &   C. H. Berry

Southern Branch
of the
University of California
Los Angeles

THE AMERICAN SOCIETY OF
MECHANICAL ENGINEERS

29 WEST THIRTY-NINTH STREET, NEW YORK

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By C. F. Hirshfeld[1] and C. H. Berry,[2] Detroit, Mich.

Members of the Society

A careful study of manufactured articles shows that even when sizes are determined by utility or use value, the choice of size is largely arbitrary. It is therefore obvious that if certain numerical values are universally accepted as preferred values, and if they are so spaced and of such extent as to fit in with all requirements met in deciding on sizes to be used, the arbitrary choices may be so made as to yield sizes expressible in terms of these preferred numbers.

Preferred numbers have been successfully used in Germany and some other European countries, and this paper is presented as a background for a study of the subject in connection with American conditions and problems. The authors develop preferred-number systems based on the theory of geometrical series and apply them to various industries. Graphs and tables illustrate the points brought out.

SIZE figures in one way or another in all manufactured articles and, in fact, in all articles of commerce. For present purposes the word "size" must be interpreted in its broadest possible sense. It may indicate any one of the following specifications: a purely arbitrary size, such as Model No. 1, Model No. 2, 3, etc., of a given line of manufactured article; a conventional size upon which all manufacturers in a given line have agreed, as sizes of hats or shoes; the weight of a package in which a given material is sold; the weight of some arbitrarily chosen quantity, as 100-lb. rails or 10-oz. duck; an actual or conventional significant dimension, as 1-in. round stock or 1-in. lumber; or any one of the numerous dimensions which may be required in the design, fabrication, or marketing of a given article or manufactured product.

Viewed from one aspect, size is second only to the product itself when dealing with the materialistic side of manufacture and commerce. All manufacture and all commerce are carried on in terms

of size. In fact, the need for means of expressing size is probably the underlying reason for the heterogeneously assorted systems of expression now in use.

Further, size, in the general sense of the word and also in the specific sense, is directly or indirectly made up of two components or factors. One of these is a number and the other a dimension, as in "1-in." bar stock.

Numerous cases will be found in which this composite structure is not evident, but it will always appear if the search is carried far enough. When a given kind of equipment is sold under the size designations, Model No. 1, Model No. 2, etc., the model size itself is determined by its physical size or capacity. Thus Model No. 1 may be 4 ft. high, or may have a capacity of 1 ton in a given time, while Model No. 2 may be 6 ft. high or have a capacity of 2 tons, etc.

In any general study of sizes we must therefore consider two components which are respectively numerals and units of measurement. These units of measurement are matters of custom or use which vary both with the type of measurement and with the systems adopted in different countries.

In considering the so-called "Preferred Numbers" we have no direct interest in these dimensional units. Our concern is entirely with the numerical part of the doublet used for expressing size, except in so far as the relations between dimensions in any one system of measurement may make certain numerals more or less convenient. As an example of the significance of this exception, consider the possible means of expressing a length of 27 ft. in the English system. This might be called 9 yd., 27 ft., or 324 in. In measuring dress goods the yard is the most convenient or at least the conventional unit and thus brings about the necessity for the use of the numeral 9 in expressing the size indicated. In certain other types of measurement, as, for instance, in measuring length of lumber, the foot unit is the most convenient and thus makes necessary the use of the numeral 27. The inconvenience of using the number 324 probably explains the use of feet and yards instead of inches for large measurements.

Sizes used in industry and commerce may be divided into several different categories or classes, as follows:

1 Sizes which are entirely matters of style, such as the lengths of coats, the heights of hats, and other dimensions which change from year to year.

2 Sizes which are determined entirely by personal comfort, such as the sizes of men's collars, the sizes of hats and shoes, etc. Each of these series of sizes has been worked out by experience and it is not unreasonable to assume that at the present time they form satisfactory systems.

3 Sizes which are entirely matters of taste, though not necessarily matters of fashion. Thus the proportions of a Doric column enter-

ing into a structure are not determined by strength but by appearance. Proportions or sizes of furniture, objects of art, and many architectural features fall in this class.

4 Sizes which are determined by a combination of appearance and utility. The sizes of drawer pulls, door knobs and the like fall into this class, but for the present they are outside the scope of this paper though they may later fall partly or wholly within it.

5 Sizes which are determined entirely by utility or use value.

Class 1 sizes are by nature arbitrary and changeable and those of classes 2, 3, and 4 are outside the scope of the present paper. Such sizes as are grouped under class 5, whether they refer to buckets and pails, pots and kettles, bolts, wires, or to any of the innumerable machine parts, fall within the scope of what may be called the Theory of Preferred Numbers.

With the preceding paragraphs by way of introduction, something may now be said with respect to this theory, what it is, and what it is for.

THE THEORY OF PREFERRED NUMBERS

At the present time there is much that is arbitrary in the choice of size, even when size is determined by utility or use value, and careful study will show that slight variations in the sizes finally decided upon would not make a great difference in the use value of the pieces. For example, have we any proof that pails should be made in 8-, 10-, 12-, and 14-qt. sizes instead of in, say, 9-, 11-, 13- and 15-qt. sizes? Or again, when we calculate the required diameter of a circular section in a piece of machinery to be 2.237 in., are we justified in assuming our calculation so accurate that 2.25 or 2.2 in., or possibly even 2 or 2.5 in., will not prove equally suitable?

Careful study in any drafting room will show that the latitude of choice allowed the designer is such that his decision with respect to final dimensions is arbitrary within certain limits, and quite often within very wide limits.

In view of these facts it is quite obvious that if certain numerical values are universally accepted as preferred values and if they are so spaced and of such extent as to fit in with all requirements met in deciding on sizes to be used, the arbitrary choices may be so made as to yield sizes expressible in terms of these preferred numbers. Moreover, if such a thing is possible, very material savings should result from its use, some of the most obvious of which are:

a Mill products which are used in the fabrication of manufactured articles could be made in a minimum number of standard sizes so chosen as to meet the needs of users who have adopted preferred numbers for the sizes of their wares.

b Measuring instruments and production machinery might be simplified and cheapened, because it would be necessary to provide for their use with preferred dimensions only instead of providing for universal adjustment.

c Odd sizes, manufactured through ignorance of real requirements or to meet the supposed, but really illogical, needs of a customer or industry, might be eliminated.

d Life would be made simpler for both the producers and the users, because calculation, manufacture, commerce, catalogs, price lists, and human memory would deal only with certain easily memorized and widely used numerals. Certain other advantages will become apparent as the subject is further developed.

One of a practical turn of mind is likely to think that there is much of theory in all this and little of practical worth. Sizes have been developed by a cut-and-try process and with commercial necessity acting as a brake on the overdevelopment of sizes. It might be assumed, therefore, that present-day industry is using the minimum number of sizes consistent with the meeting of human needs and that these sizes are most advantageously chosen. Such arguments are weighty and worthy of serious consideration.

However, systems of preferred numbers have been accepted to a certain extent in Germany, and several other European countries are indicating an intention of following this lead after having made a study of the German systems. It would seem, therefore, that we should not complacently accept our present methods and practices, but instead give serious thought and study to this problem.

The authors of this paper are not urging the adoption of a system or systems of preferred numbers, but they do urge most emphatically a study of the subject in connection with American conditions and problems to the end that decision for or against may be made with full knowledge of all that is involved. Such a study may probably be most conveniently undertaken by considering sizes actually in use in this country and the relation which such sizes bear to a possible series or to several possible series of preferred numbers. Some examples of common cases are given in the paragraphs immediately following.

Common wire nails are sold in sizes expressed in terms of "pennies;" thus we have 2d nails, 3d nails, and so on up to 80d nails. These designations originally indicated the price per hundred, but now, by arbitrary agreement, they represent certain lengths which are expressed in even inches and fractions of inches which have no connection whatever with the number used in expressing the commercial size. The diameters of stock from which the nails are made vary from a small value at the low end of the series to a large value at the upper end. Presumably the nail is intentionally or unintentionally proportioned as a long column, and there is therefore probably some approximation to a definite relation between diameter and length.

The values of length and diameter for wire nails are plotted against the commercial sizes in Fig. 1, from which it can be seen that wire-gage sizes and lengths of wire nails are not as well coordinated as they might be. The nearest convenient commercial

size of wire has been combined with the desired length to give a certain size of nail. In fact, half and quarter gage sizes have been used in some cases. The effect of this is indicated by the jagged line showing strength plotted against size. It is obvious that the use of one size of wire for two successive sizes of nails results in most erratic variations in strength. The nails are of course commercially satisfactory, but this does not mean that they are constructed with the minimum use of material or that a more satisfactory line of nails might not be developed. This matter will be considered later after certain other matters have been discussed.

Wire is sold in certain sizes specified as "gages." Copper wire is measured in terms of the American or Brown and Sharpe Gage.
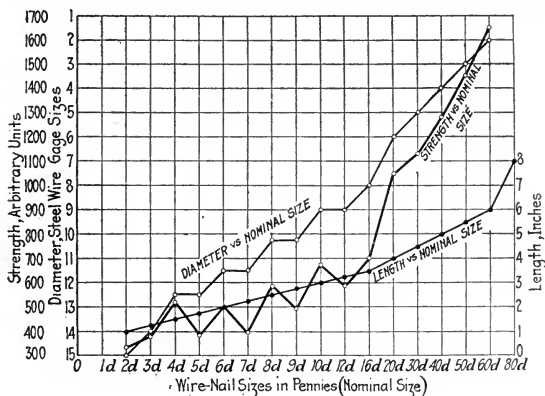
FIG. 1 COMMERCIAL SIZES OF WIRE NAILS

The diameters corresponding to successive gage sizes are plotted to semi-logarithmic coördinates in Fig. 2 and it is apparent that this series of sizes was developed according to a definite and consistent plan.

Steel wire is measured in terms of the Washburn and Moen, Roebling, or Steel Wire Gage. The diameters corresponding to successive gage sizes are also plotted in Fig. 2. Evidently there is some underlying plan, but it follows what appear to be imperfect laws. Apparently a certain law of variation is followed until it cannot be followed further. Another is then chosen but later abandoned, and so on. There may be good reasons back of this peculiar variation, but it seems probable that this gage might have been built up upon a much simpler basis. More will be said about this in later paragraphs.

As another example of sizes in commercial use we may consider frying pans. The diameters of one well-known make of frying pans are plotted against commercial or nominal sizes in Figs. 3 and 4. This matter will also be considered later.
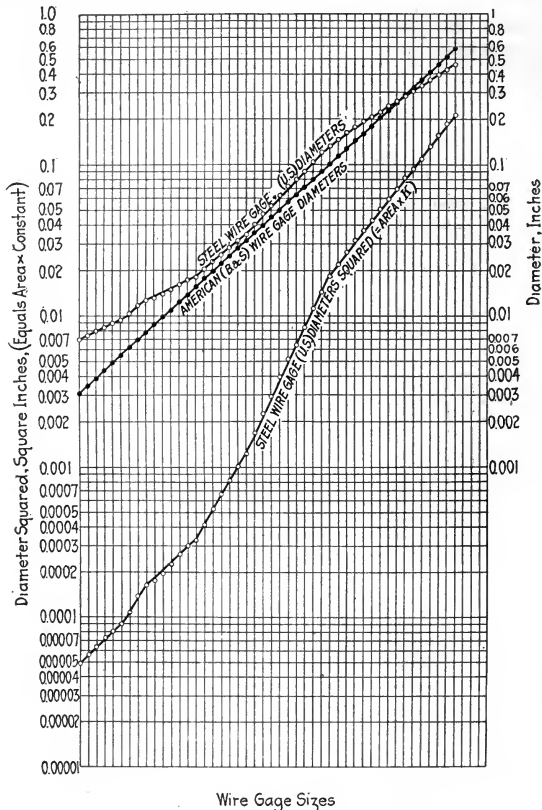
Wire Gage Sizes

FIG. 2   SEMI-LOGARITHMIC PLOTTING OF WIRE-GAGE SIZES

Saucepans and preserving kettles are sold in sizes designated in terms of quarts. Saucepans are made in comparatively small sizes and preserving kettles in comparatively large sizes, but the
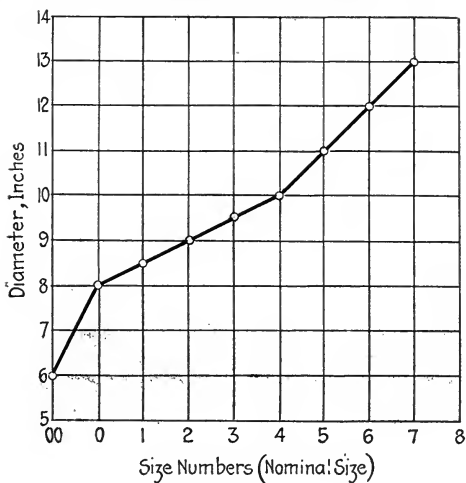
FIG. 3
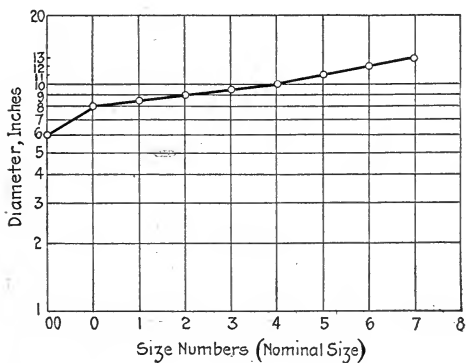
FIG. 4

FIGS. 3 AND 4   DIAMETERS OF FRYING PANS PLOTTED AGAINST COMMERCIAL
OR NOMINAL SIZES
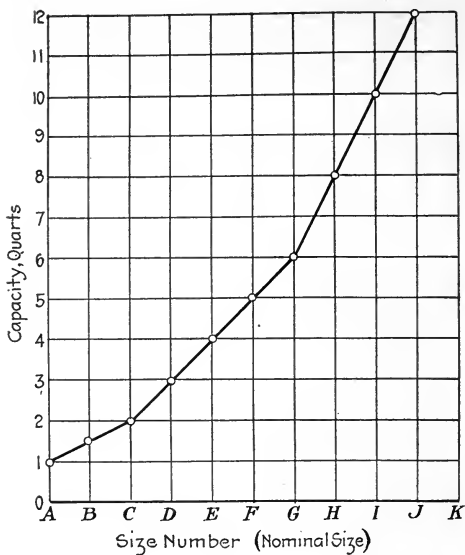
FIG. 5
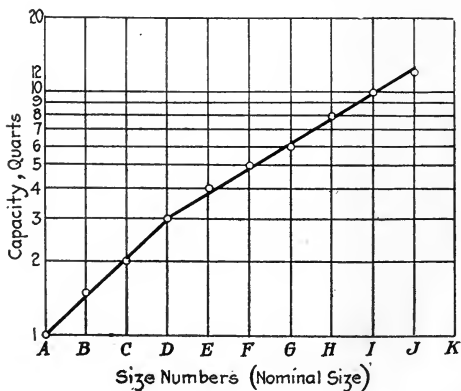
FIG. 6

FIGS. 5 AND 6 CAPACITIES OF SAUCEPANS PLOTTED AGAINST COMMERCIAL
OR NOMINAL SIZES

two overlap so that sizes of equal capacity can be obtained in the larger sizes of saucepans and in the smaller sizes of preserving kettles. The two kinds of utensils are interchangeable in use, so that it is probably not illogical to consider them together. The sizes available in one make are plotted to ordinary coördinates in Fig. 5.
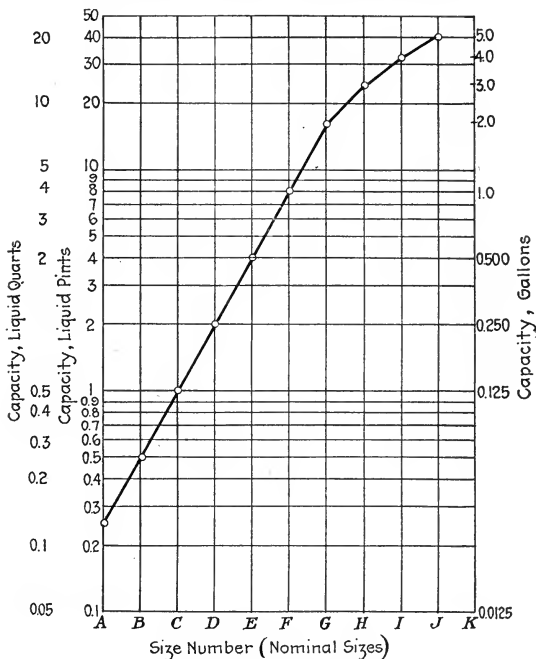
Fig. 7 Capacities of Liquid Measures Plotted against Nominal Sizes

The same values are plotted to semi-logarithmic coördinates in Fig. 6.

Liquid measures are available in numerous sizes, all commonly designated in terms of capacity. The sizes obtainable from one well-known maker are plotted to semi-logarithmic coördinates in Fig. 7.

Commercial sizes of steel shafting are shown in Fig. 8.

Bolts have been standardized in different ways and for different

purposes by various organizations. One very complete standard is that of the Society of Automotive Engineers. The diameter, area of stock, and area at root of thread for S.A.E. Standard bolts are plotted to semi-logarithmic coördinates in Fig. 9. A similar set of graphs for the United States Standard is given in Fig. 10.

Pipes are well standardized in several different weights. They are sold in terms of nominal diameters and the actual diameters practically never equal the nominal diameters. The actual inside diameters and areas of standard full-weight wrought-iron pipes
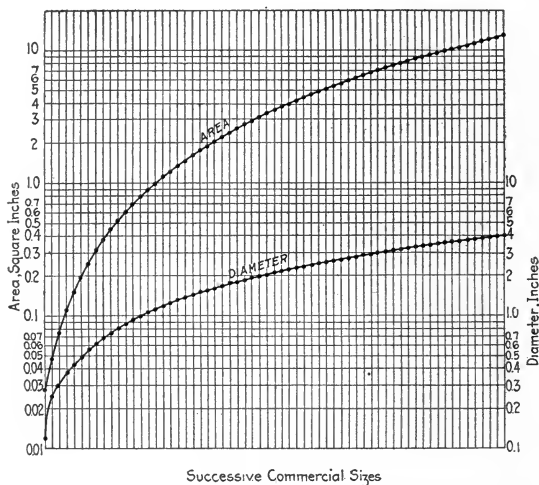
Fig. 8   Diameters and Areas of Steel Shafting Plotted against Successive Commercial Sizes

from $^1/_8$ in. to 10 in. nominal diameter are plotted to semi-logarithmic coördinates in Fig. 11.

A brief survey of Figs. 2, 4, 6, 7, 8, 9, 10 and 11, all of which show sizes plotted to semi-logarithmic coördinates, will indicate a surprisingly large number of cases in which successive sizes fall very nearly on one straight line or on two or three straight lines of slightly different slopes. If space permitted many more examples of the same thing could be given.

This peculiar tendency naturally suggests some underlying law of size or size variation. If sizes tend to fall on such straight lines, it follows that the mathematical characteristics of such lines must express what we may call the natural law of size variation.

The Application of the Theory of Geometrical Series

On semi-logarithmic paper a straight line which is not parallel to either axis is the plot of a geometrical series. That is, such a line represents a succession of terms each of which bears a constant ratio to the preceding one. The following succession of terms represents part of a geometric series:

$$a, 2a, 4a, 8a, 16a, \ldots \ldots \ldots \ldots \text{ etc.}$$
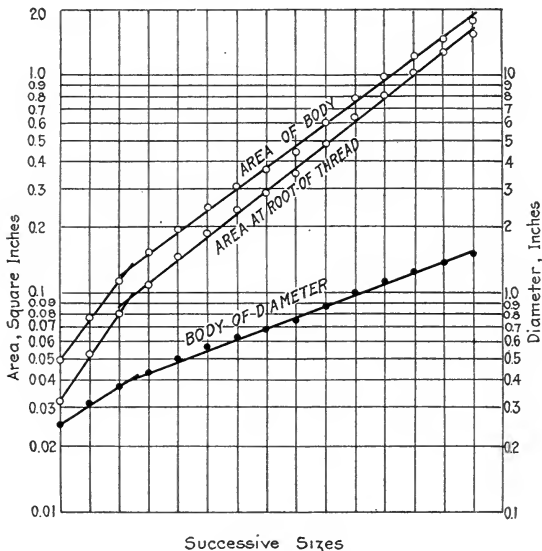
Fig. 9 S.A.E. Standard Bolts

Such a series can be written symbolically, as:

$$a, ra, r^2a, r^3a, \ldots \ldots \ldots \ldots \ldots r^{n-1}a$$

in which—

  a = first term of the series or the number on which it is built up
  r = ratio of each term to the preceding term, and
  n = the number of the term in the series.

German scientists and technicians have investigated this matter of size variation, including several geometrical series which may be used as expressions of the law of size variation. While they have not yet formally adopted them, they are leaning very markedly

toward a system of series with ratios equal to the 5th, 10th, 20th, 40th, and 80th roots of 10 and have, in fact, adopted it tentatively.
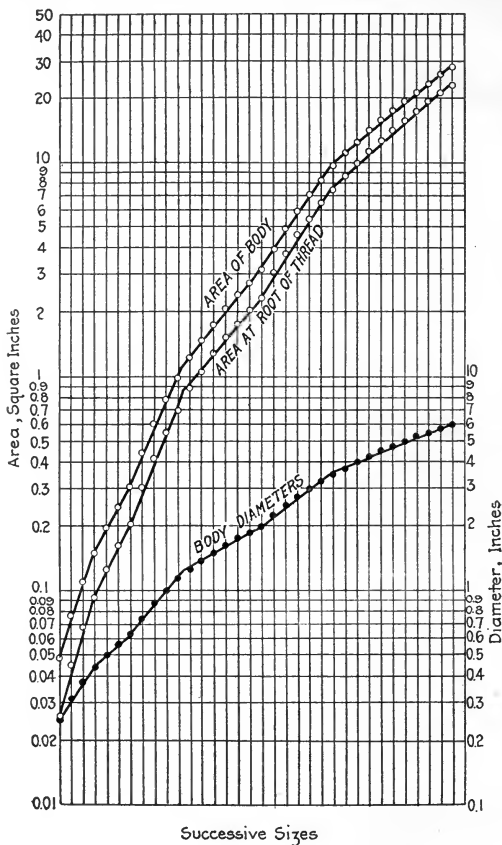
Fig. 10   U. S. Standard Bolts

Such series would be simply expressed as follows for the 10th-root series:

$$a, \sqrt[10]{10}\ a, (\sqrt[10]{10})^2\ a, (\sqrt[10]{10})^3\ a, \text{ etc.}$$

which is equal to—

$$a, 1.259a, 1.58a, 1.99a, \text{etc.}$$

For the 20th-root series:

$$a, \sqrt[20]{10}a, (\sqrt[20]{10})^2 a, (\sqrt[20]{10})^3 a, \text{etc.}$$
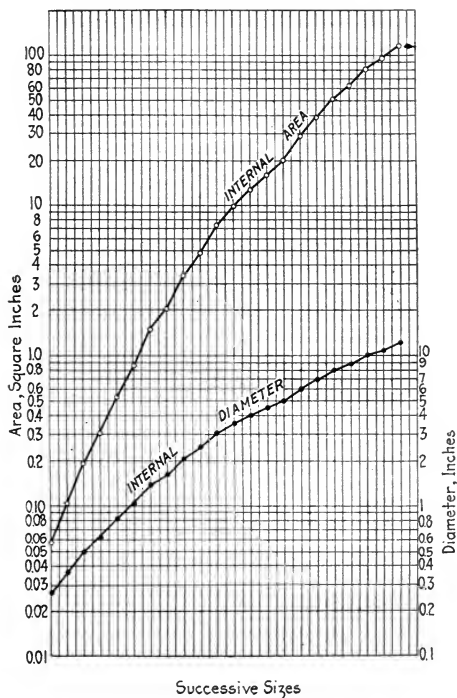
FIG. 11   STANDARD WROUGHT-IRON PIPE

which is equal to—

$$a, 1.122a, 1.259a, 1.41a, \text{etc.}$$

The whole idea can be illustrated most clearly by dealing with the series which has a ratio equal to $\sqrt[100]{10}$, i.e., $10^{1/100}$. Such a series built upon the number 1, that is, with a equal to 1, is drawn to semi-logarithmic coördinates in Fig. 12.

Assume that it is desired to make a given article in eleven sizes progressing as in this series. The size designations are indicated as A, B, C, D, etc., in Fig. 12. The dimensional sizes are given by the values of the ordinates above the size designations. If a smaller number of models equally distributed over the same range of dimensional sizes is required, we might use sizes A, C, E, G, I, and K. But such a choice would be exactly equivalent to using a series such as that shown at the left in Fig. 13, which is a series with the ratio $\sqrt[50]{10}$. Or if we desired only three sizes say, A, F,
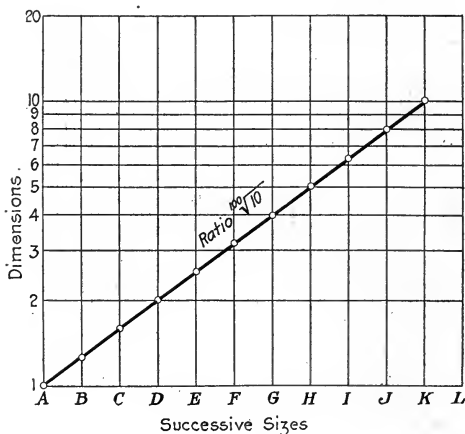
Fig. 12   Graph of Size Series with Ratio $\sqrt[100]{10}$

and K, this would be equivalent to using the $\sqrt[20]{10}$, or a series such as that shown to the right of Fig. 13.

Bearing these ideas in mind, inspection of Fig. 14 will show that a series with ratio $\sqrt[10]{10}$ makes available only two of the sizes here under consideration, namely, the first and the last. A series with ratio $\sqrt[20]{10}$ gives three sizes, the same two extremes and one intermediate corresponding to F of the original arrangement. A series with ratio $\sqrt[30]{10}$ yields four sizes, the two intermediates not corresponding to previous sizes because 30 is not evenly divisible into 100. A proportionate number of sizes is obtainable with the series with ratios equal to $\sqrt[40]{10}$, $\sqrt[50]{10}$, $\sqrt[70]{10}$, $\sqrt[80]{10}$, and $\sqrt[90]{10}$. The series with ratio equal to $\sqrt[50]{10}$ yields six of the original sizes as previously indicated.

Going back now to the original series with ratio equal to $\sqrt[100]{10}$, shown in Fig. 12, let us assume that the 11 sizes designated by letters A to K, inclusive, are not sufficient, that is, the finer subdivisions are required. These can be obtained by inserting sizes midway between A and B, B and C, C and D, etc., as shown in Fig. 15. We should then get sizes which might be designated as A, $A^1/_2$, B, $B^1/_2$, C, $C^1/_2$, etc., and the dimensions of each size would be related to the dimension of the preceding size by the same ratio, but this ratio would not be the same as that in the case first considered.

The last statement is likely to be a bit puzzling to those who do
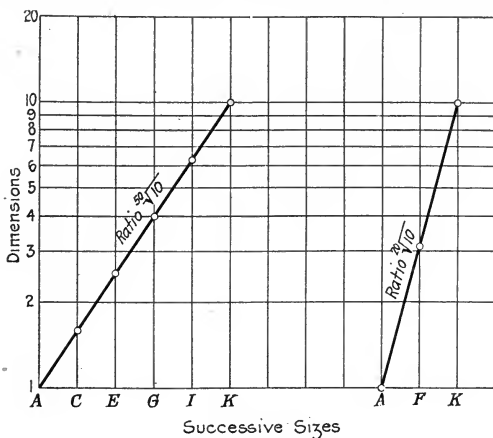
FIG. 13   GRAPHS OF SIZE SERIES HAVING RATIOS $\sqrt[50]{10}$ AND $\sqrt[20]{10}$

not handle such series frequently, but it is easily explained. The line as drawn yields a value of 10 as ordinate for 100 of the smallest divisions on the horizontal axis. But 10 is the 100th power of $\sqrt[100]{10}$. The ordinate of the first small division counting from the left on the horizontal axis is therefore equal to the ordinate at A multiplied by $\sqrt[100]{10}$; the ordinate of the second small horizontal division is the ordinate at A multiplied by $\sqrt[100]{10} \times \sqrt[100]{10}$, or $(\sqrt[100]{10})^2$; the ordinate at B is equal to the ordinate at A multiplied by $(\sqrt[100]{10})^{10}$; the ordinate at C is equal to the ordinate at A multiplied by $(\sqrt[100]{10})^{20}$; and so on. It is therefore obvious that—

Ordinate at B = Ordinate at $A \times (\sqrt[100]{10})^{10}$

Ordinate at C = Ordinate at $B \times (\sqrt[100]{10})^{10}$, etc.

but—

$$\text{Ordinate at } A^{1}/_{2} = \text{Ordinate at } A \times (\sqrt[100]{10})^{5}, \text{ and}$$

$$\text{Ordinate at } B = \text{Ordinate at } A^{1}/_{2} \times (\sqrt[100]{10})^{5}.$$

That is, the ratio between successive sizes of the A, B, C, D, etc., series is $(\sqrt[100]{10})^{10} = \sqrt[10]{10}$, and the ratio between successive sizes of the $A^{1}/_{2}$, A, B, $B^{1}/_{2}$, etc., series is $(\sqrt[100]{10})^{5} = \sqrt[20]{10}$. If desired, the subdivision can be carried to any extent, but no matter how far it is carried the same characteristic relations will hold true.
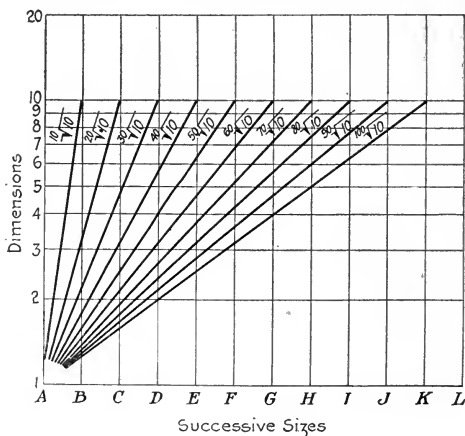
Fig. 14 Sizes Available when Series with Different Ratios are Employed

If the requirements to be met are such that comparatively small variations of size are required in the smaller sizes and larger variations in the larger sizes, we might choose an arrangement such as that shown in Fig. 16. This appears to be a very minor variation of what has preceded, but in fact it is quite a major variation. The series of sizes resulting from such uneven subdivision no longer has the same characteristics as were described in connection with the sizes A, B, C, D, etc. The ratio between successive sizes changes at E, H, I, and J. This is shown clearly by plotting dimensions from Fig. 16 with nominal sizes evenly spaced as in Fig. 17. The ratios are indicated on each part of the broken line of this figure.

From what has been done in developing the graph in Fig. 16, it is apparent that its shape will vary with the way in which one chooses to distribute nominal sizes among the possible evenly dis-
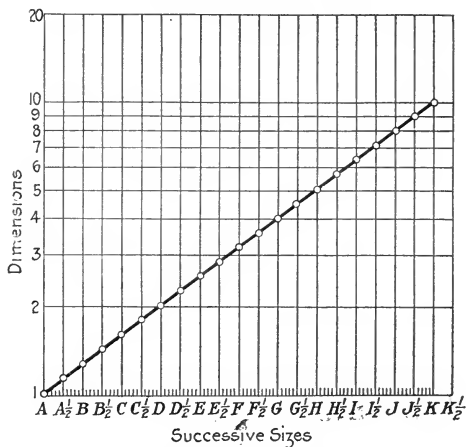
FIG. 15 SHOWING METHOD OF INTERPOLATING SIZES IN A SERIES
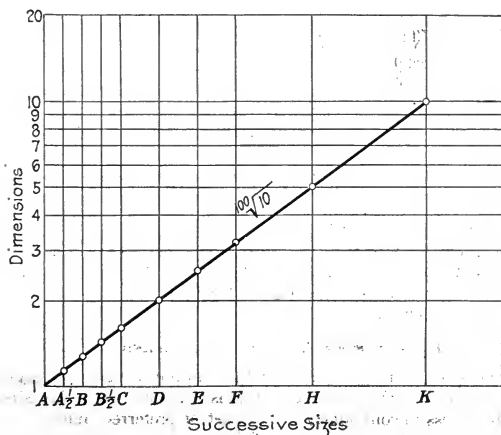
FIG. 16 UNEVEN SUBDIVISION OF SIZES

tributed sizes. If the nominal sizes be closely spaced with reference to possible sizes with any assumed even increments, the resultant line will be less steep than if the nominal sizes are more widely spaced among the possible sizes.

Comparison of Figs. 12 to 17, inclusive, with Figs. 2, 4, and 6 to 11, inclusive, will show sufficient resemblance between the two groups to lead one to suspect that possibly the geometrical series is an expression of the law which underlies variation of size in articles whose size is determined by use value. One may object to such a conclusion by pointing out that in many cases in which actual sizes
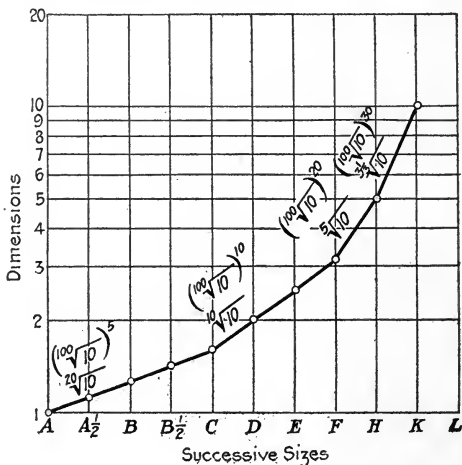
Fig. 17   Dimensions from Fig. 16 Plotted with Nominal Sizes Evenly Spaced

are plotted they do not fall exactly on the straight lines which have been drawn, and, further, that the use of a logarithmic scale distorts the significance of vertical departure from the lines so that it is possible that a small departure on the plot may mean a large discrepancy in actual size.

Present System Similar to Preferred Numbers

These criticisms are valid but they do not vitiate the suggested conclusion. Rather peculiarly, this is true because we are already more or less unconsciously using a set of preferred numbers. We use certain fractions of inches for dimensions smaller than one inch in preference to all other possible fractions. Thus our first choice

is $1/2$ in.; the next, $1/4$ and $3/4$ in.; the next $1/8$, $3/8$, $5/8$, and so on. For dimensions larger than one inch we do the same sort of thing but in two different ways. Our choice with respect to fractions of one inch is the same as before, namely, a system based on eight with preference given to $4/8$; $2/8$, $4/8$, and $6/8$; $1/8$, $2/8$, $3/8$, $4/8$, $5/8$, $6/8$, and $7/8$; etc., in the order indicated by groups. But for fractions of one foot we use a system based on twelve and, whether we express ourselves in inches or in fractions of a foot, we give preference to $6/12$; $3/12$, $6/12$, $9/12$; $2/12$, $4/12$, $6/12$, $8/12$, $10/12$; etc., in the order indicated by groups.

Therefore, in actually setting the dimensions which shall characterize different sizes of manufactured products, we generally
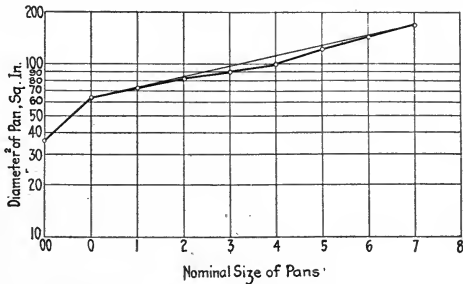
FIG. 18 SQUARES OF DIAMETERS OF FRYING PANS PLOTTED AGAINST
NOMINAL SIZES

choose on the basis of these preferred numbers, using that one which happens to fall nearest to our desire in each case.

Consider, for example, the dimensions of frying pans as plotted in Fig. 3. Diameter is certainly the most significant dimension of a frying pan. After that would come depth, and after that the combination of dimensions which affect ability to stack different sizes or a number of the same size. The smallest diameter is 6 in., a very convenient number, actually the preferred $6/12$ or $1/2$. This size is used for such purposes as frying a single egg, a single chop, or other small quantities of food. The next size has a diameter of 8 in., another one of our unconsciously preferred numbers, and beyond that diameters increase by half-inches to a diameter of 10 in. Beyond this size the diameters increase by 1-in. increments.

If there is any significance to the diameter of a frying pan it must result from the fact that the area is proportional to the square of that dimension. The squares of the diameters, and therefore figures proportional to cooking area, are plotted against nominal sizes in Fig. 18. The most casual inspection shows that in all probability no law underlies the choice of these diameters, and one

is driven to the conclusion that sizes of about the proportions made have been found useful and that they have been chosen to fall on the preferred half-inch and whole-inch intervals.

Let us assume for the sake of argument that a 6-in., an 8-in., and a 13-in. size are necessary and that the cooking area is the underlying, governing criterion in choice of size. Let us also assume that six sizes are required between the 8-in. and the 13-in. sizes. The simplest possible arrangement would result if we used diameters consistent with a straight line drawn through the 8-in. and 13-in. points in Fig. 18. The resultant areas and diameters are given in Table 1 for ready comparison with the actual commercial diameters.

TABLE 1 COMPARISON OF CALCULATED AND COMMERCIAL
DIAMETERS OF FRYING PANS

Nominal Size	Area	Diameter	Commercial Diameter
0	64.0	8.00	8.0
1	73.5	8.56	8.5
2	84.0	9.16	9.0
3	96.5	9.83	9.5
4	111.0	10.53	10.0
5	128.0	11.31	11.0
6	147.0	12.12	12.0
7	169.0	13.00	13.0

It is at once apparent that the diameters corresponding to the smooth line of Fig. 18 are most inconvenient numerical values in comparison with the more commonly used numerical values which represent the actual commercial diameters. On the other hand, it is probably true that the resulting sizes are far more rational and would prove of greater use value, if there were any means of measuring such use value, since each size bears a certain definite constant relation to that which precedes it and that which follows it in the series. It is possible that a still better result could be obtained by using two different slopes between the two extremes so that diameters increased more slowly at the start and more rapidly later. Such an arrangement would correspond more nearly to present commercial sizes.

Now suppose for a moment that instead of using such values as even inches and half-inches we had at some time arbitrarily decided to give preference to, let us say, the curious numbers 8, 8.5, 9.2, 9.8, 10.5, 11.3, 12.1 and 13. If, under those circumstances, we developed a set of diameters for frying pans as given in the third column of Table 1 we should have driven ourselves into the situation illustrated by Table 2.

TABLE 2 COMPARISON OF CALCULATED AND PREFERRED
DIAMETERS OF FRYING PANS

Calculated Diameter	Preferred Numbers
8.00	8.0
8.56	8.5
9.16	9.2
9.83	9.8
10.53	10.5
11.31	11.3
12.12	12.1
13.00	13.0

Obviously, we would not hesitate to round out the calculated values to the preferred values, and the frying pans built upon the preferred-number diameters would probably have just as great a use value as would pans built upon the calculated diameters.

As another example of the use of preferred numbers, consider the matter of wrought-iron and steel pipe sizes. This material is sold in several different "weights" or wall thicknesses, additional thickness being obtained in most cases at the expense of internal diameter. In order to simplify matters to the greatest possible extent only one weight will be considered, namely, Standard Full Weight, and only such sizes will be used as are commonly sold on the basis of nominal inside diameter, that is, sizes up to and including 12-in. pipe. The sizes in which such pipe is graded are designated in inches and fractions of inches and refer to a nominal inside diameter, not to a real diameter. Table 3 will indicate the extent to which the nominal diameter is purely a fictitious designation.

TABLE 3   NOMINAL AND APPROXIMATE ACTUAL PIPE SIZES

Nominal Diameter, in.	Decimal Equivalent of Nominal Diameter, in.	Approximate Internal Diameter, in.
$1/8$	0.125	0.269
$1/4$	0.250	0.364
$3/8$	0.375	0.493
$1/2$	0.500	0.622
$3/4$	0.750	0.824
1	1.000	1.049
$1^1/4$	1.250	1.380
$1^1/2$	1.500	1.610
2	2.000	2.067
$2^1/2$	2.500	2.469
3	3.000	3.068
$3^1/2$	3.500	3.548
4	4.000	4.026
....	.....	.....
....	.....	.....
....	.....	.....
10	10.000	10.192
11	11.000	11.000
12	12.000	12.090

It is of interest to note that while we purchase such pipe in terms of even inches and fractions of inches, what we really purchase is pipe having internal diameters designated by figures which are apparently just as inconvenient as any of those which were worked out for frying-pan diameters. We are thus in fact using figures of this same form and sort but disguising them in order to express sizes in our preferred-number system.

Pipes are used principally as conveyors of liquids and gases and the significant dimension is the internal diameter, since this determines the cross-sectional area and therefore the carrying capacity of the pipe. Inspection of Fig. 11 shows a rather ordered progress of cross-sectional area from the smallest to the largest pipe sizes. It is probable that there was originally some reason back of the variations from exact order in successive sizes and areas. Possibly it had something to do with thickness of material available, or with some necessary relation between internal and external diameters,

or, more remotely, it may have had something to do with strength against bursting.

However, it should certainly be possible to make pipe to areas such as those indicated by the straight lines drawn on Figure 19. If this were done, the actual internal diameters would be as indicated
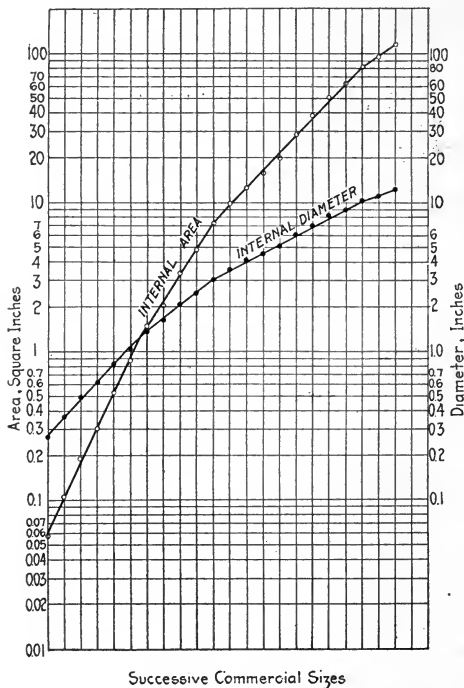
FIG. 19   STANDARD WROUGHT-IRON PIPE
(Points represent actual diameters and areas as now made.)

by the other series of lines on the same figure.   Inspection of the locations of the points representing the actual present-day diameters with reference to their proximity to the new-diameter lines will show that the new diameters required will not vary greatly from those actually in use.   The variation appears to be of a much smaller order than was found in the study of frying pans.

If one objected to designating pipes in terms of these odd actual

internal diameters, it would be perfectly possible to use the same sort of fiction which we now use under exactly the same conditions.

Examples of these sorts could be produced in almost endless variety, but it seems as though those just cited are sufficient to illustrate the points under discussion. We do use preferred numbers now, and very often these preferred numbers are used in a purely nominal sense while the real size dimensions are expressed in unwieldly decimal fractions. In some cases this is so extreme that we give these inconvenient decimal fractions special designations so that we will not have to deal with the numbers themselves. Such a case has just been considered in connection with pipes. Many others exist. Thus a No. 22 copper wire means something
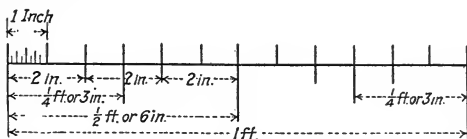
Fig. 20 Length of One Foot so Subdivided That Lengths of Division Lines Represent Order of Preference

to almost every engineer, and yet there are very few who know that a No. 22 copper wire has a diameter of 0.0253 in. And why should they when the convenient round number 22 meets all ordinary needs and when exact diameter and area can always be obtained from a gage table if needed?

Unfortunately our preferred numbers are most irrationally related from many points of view. This follows directly from our conventional use of the inch and foot and from the way in which our system of numbers is built up. The effect of the inch and foot is illustrated in Fig. 20, in which a length of 12 in. is laid out to an arbitrary scale with certain preferred subdivisions set in in such a way that the length of the subdivision line represents the order of preference. It is apparent that any series of sizes which starts with dimensions less than one inch and ends with dimensions expressed in feet, inches, and fractions of an inch must almost certainly be made up of steps of most varying mathematical character.

Construction of Geometrical Series

Again, our system of numbers is itself basically peculiar. The logical steps of progression from 1 to 10 are by units or by simple or fractional multiples of units. The tendency is always toward even steps and there are surprisingly few possibilities. Thus we may use the following:

1, 10; 1, 5, 10; (1, 2.5, 5, 7.5, 10);
1, 2, 4, 6, 8, 10; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

These series are all arithmetical, and any attempt to arrange a geometrical series between 1 and 10 leads immediately to complicated, or at least inconvenient, decimal fractions.

Above 10, conditions are somewhat better in that there are numerous geometrical series yielding simple numbers, such as—

> 10, 20, 40, 80, 160, 320, etc.
> 10, 30, 90, 270, 810, etc.
> 10, 40, 160, 640, etc.
> 10, 50, 250, 1250, etc.

but it is obvious that the increase of value in the first series, which is characterized by the slowest rate of increase, is exceedingly rapid.

With the exception of such geometrical series one is limited to arithmetical series in developing steps of such character as to yield simple even numerals. One may progress by ones, twos, threes, fours, fives, etc., up to any desired value and thus develop any number of arithmetical series. But any attempt to develop geometrical series other than those just indicated leads immediately to fractional numbers.

These are the facts with which we are confronted, and, even if it be assumed that we can and possibly may change our system of measurement, it seems foolish to suppose that we will ever change our system of numbers. It looks, therefore, as though we may as well make up our minds to do the best we can with what we have.

The Germans have an official system of measurement which coincides in arrangement with the system of numbers. They therefore seem to have only one problem to solve where we have two. The truth of the matter is that in many cases they also have two, but there are enough cases in which they have only the one so that they can confine their attention largely to those cases for the time being. The movement to adopt preferred numbers may be regarded as an effort to evolve a number system better suited to our technical needs than the present system. Of necessity it is expressed in terms of our present system and as a result it appears at a disadvantage.

The Germans have two sets of preferred numbers and, the authors are informed, these numbers are now in use to a limited extent. The first is a set of "standard diameters," and was adopted before the recent development of the more general "preferred numbers." The second is built up on the 80th root of 10, and is like the one discussed in connection with Figs. 12 to 17. The method of construction is best illustrated by writing down the numerical terms of such series. These values are given in Table 4 for all series between that with ratio $\sqrt[5]{10}$ and that with ratio $\sqrt[80]{10}$.

A casual inspection of Table 4 shows that the series with ratio $\sqrt[10]{10}$ contains all the terms comprised in the series with ratio $\sqrt[5]{10}$ and in addition one term which is the geometrical mean between every two of those in the simpler series. Similarly, the series

TABLE 4 PREFERRED-NUMBER SYSTEM ADOPTED IN GERMANY—EXACT VALUES

Ratio = $\sqrt[5]{10} = 1.585$		Ratio = $\sqrt[10]{10} = 1.259$		Ratio = $\sqrt[20]{10} = 1.122$		Ratio = $\sqrt[40]{10} = 1.059$		Ratio = $\sqrt[80]{10} = 1.029$	
Term	Numerical Value	Term	Numerical Value	Term	Numerical Value	Term	Numerical Value	Term	Numerical Value
0	1.000	0	1.000	0	1.000	0	1.000	0	1.000
..	....	..	....	..	....	..	....	1	1.029
..	....	..	....	..	....	1	1.059	2	1.059
..	....	..	....	..	....	..	....	3	1.090
..	....	..	....	1	1.122	2	1.122	4	1.122
..	....	..	....	..	....	..	....	5	1.155
..	....	..	....	..	....	3	1.189	6	1.189
..	....	..	....	..	....	..	....	7	1.223
..	....	1	1.259	2	1.259	4	1.259	8	1.259
..	....	..	....	..	....	..	....	9	1.296
..	....	..	....	..	....	5	1.334	10	1.334
..	....	..	....	..	....	..	....	11	1.373
..	....	..	....	3	1.413	6	1.413	12	1.413
..	....	..	....	..	....	..	....	13	1.454
..	....	..	....	..	....	7	1.496	14	1.496
..	....	..	....	..	....	..	....	15	1.540
1	1.585	2	1.585	4	1.585	8	1.585	16	1.585
..	....	..	....	..	....	..	....	17	1.631
..	....	..	....	..	....	9	1.679	18	1.679
..	....	..	....	..	....	..	....	19	1.728
..	....	..	....	5	1.778	10	1.778	20	1.778
..	....	..	....	..	....	..	....	21	1.830
..	....	..	....	..	....	11	1.884	22	1.884
..	....	..	....	..	....	..	....	23	1.939
..	....	3	1.995	6	1.995	12	1.995	24	1.995
..	....	..	....	..	....	..	....	25	2.054
..	....	..	....	..	....	13	2.113	26	2.113
..	....	..	....	..	....	..	....	27	2.175
..	....	..	....	7	2.239	14	2.239	28	2.239
..	....	..	....	..	....	..	....	29	2.304
..	....	..	....	..	....	15	2.371	30	2.371
..	....	..	....	..	....	..	....	31	2.441
2	2.512	4	2.512	8	2.512	16	2.512	32	2.512
..	....	..	....	..	....	..	....	33	2.585
..	....	..	....	..	....	17	2.661	34	2.661
..	....	..	....	..	....	..	....	35	2.738
..	....	..	....	9	2.818	18	2.818	36	2.818
..	....	..	....	..	....	..	....	37	2.901
..	....	..	....	..	....	19	2.985	38	2.985
..	....	..	....	..	....	..	....	39	3.073
..	....	5	3.162	10	3.162	20	3.162	40	3.162
..	....	..	....	..	....	..	....	41	3.255
..	....	..	....	..	....	21	3.350	42	3.350
..	....	..	....	..	....	..	....	43	3.447
..	....	..	....	11	3.548	22	3.548	44	3.548
..	....	..	....	..	....	..	....	45	3.652
..	....	..	....	..	....	23	3.758	46	3.758
..	....	..	....	..	....	..	....	47	3.868
3	3.981	6	3.981	12	3.981	24	3.981	48	3.981
..	....	..	....	..	....	..	....	49	4.097
..	....	..	....	..	....	25	4.217	50	4.217
..	....	..	....	..	....	..	....	51	4.340
..	....	..	....	13	4.467	26	4.467	52	4.467
..	....	..	....	..	....	..	....	53	4.597
..	....	..	....	..	....	27	4.732	54	4.732
..	....	..	....	..	....	..	....	55	4.870
..	....	7	5.012	14	5.012	28	5.012	56	5.012
..	....	..	....	..	....	..	....	57	5.158
..	....	..	....	..	....	29	5.309	58	5.309
..	....	..	....	..	....	..	....	59	5.464
..	....	..	....	15	5.623	30	5.623	60	5.623
..	....	..	....	..	....	..	....	61	5.788
..	....	..	....	..	....	31	5.957	62	5.957
..	....	..	....	..	....	..	....	63	6.131
4	6.310	8	6.310	16	6.310	32	6.310	64	6.310
..	....	..	....	..	....	..	....	65	6.494
..	....	..	....	..	....	33	6.683	66	6.683
..	....	..	....	..	....	..	....	67	6.879
..	....	..	....	17	7.079	34	7.079	68	7.079
..	....	..	....	..	....	..	....	69	7.286
..	....	..	....	..	....	35	7.499	70	7.499
..	....	..	....	..	....	..	....	71	7.718
..	....	9	7.943	18	7.943	36	7.943	72	7.943
..	....	..	....	..	....	..	....	73	8.175
..	....	..	....	..	....	37	8.414	74	8.414
..	....	..	....	..	....	..	....	75	8.660
..	....	..	....	19	8.913	38	8.913	76	8.913
..	....	..	....	..	....	..	....	77	9.173
..	....	..	....	..	....	39	9.441	78	9.441
..	....	..	....	..	....	..	....	79	9.716
5	10.000	10	10.000	20	10.000	40	10.000	80	10.000

with ratio $\sqrt[20]{10}$ contains all those in the series with ratio $\sqrt[10]{10}$ with one additional term between every two of those occurring in the latter series.   The same sort of thing must hold, no matter how far one carries the construction of such series.

Obviously, if one wished to construct a set of models with a small number of steps or sizes, he would use some or all of the values in the $\sqrt[10]{10}$ series, or possibly even drop back to the $\sqrt[5]{10}$ series. If more sizes were wanted the $\sqrt[20]{10}$ series or the $\sqrt[40]{10}$ or the $\sqrt[80]{10}$ would be used as required.

This gives several sets of preferred numbers, all of which, however, belong to one family.

The Germans then take one further step and round off these preferred numbers, making 1.259 into 1.2; 1.585 into 1.6; etc.   This gives the final set or sets of preferred numbers.   Such sets are illustrated in Table 5, from which the method of construction will be obvious when studied in connection with Table 4.

TABLE 5   PREFERRED-NUMBER SYSTEM ADOPTED IN GERMANY—
SIMPLIFIED VALUES

Values from 1 to 48				Values from 50 to 500			
Series 1	Series 2	Series 3	Series 4	Series 1	Series 2	Series 3	Series 4
1	1	1	1	...	50	50	50
...	1.2	1.2	1.2	...	...	...	52
1.6	1.6	1.6	1.6	...	...	56	56
...	2	2	2	...	...	...	60
2.5	2.5	2.5	2.5	64	64	64	64
...	3	3	3	...	...	...	68
...	...	3.5	3.5	...	...	72	72
4	4	4	4	...	...	...	75
...	...	4.5	4.5	...	80	80	80
...	5	5	5	...	...	...	85
...	...	5.5	5.5	...	...	90	90
6	6	6	6	...	...	...	95
...	...	7	7	100	100	100	100
...	8	8	8	...	...	...	105
...	...	9	9	...	...	112	112
10	10	10	10	...	...	...	118
...	...	11	11	...	125	125	125
...	12	12	12	...	...	...	132
...	...	...	13	...	...	140	140
...	...	14	14	...	...	...	150
...	...	...	15	160	160	160	160
16	16	16	16	...	...	...	170
...	...	...	17	...	...	180	180
...	...	18	18	...	...	...	190
...	...	...	19	...	200	200	200
...	20	20	20	...	...	...	210
...	...	...	21	...	...	225	225
...	...	22	22	...	...	...	240
...	...	...	24	250	250	250	250
25	25	25	25	...	...	...	265
...	...	...	26	...	...	280	280
...	...	28	28	...	...	...	300
...	...	...	30	...	320	320	320
...	32	32	32	...	...	...	340
...	...	...	34	...	...	360	360
...	...	36	36	...	...	...	380
...	...	...	38	400	400	400	400
40	40	40	40	...	...	...	420
...	...	...	42	...	...	450	450
...	...	45	45	...	...	...	480
...	...	...	48	...	500	500	500

The authors believe that it may be a mistake to round out the numbers in these tables instead of preserving the original values. The original values are the exact values, and the extent of rounding

in some cases is so great as to entirely mask the original value. If one regards the adoption of preferred numbers as the adoption of a new number system, much can be said in favor of preserving the peculiar decimal fractions. However, only experience can prove the correctness or incorrectness of such practice.

Application of Preferred-Numbers System to Units of Measurement

Any attempt to apply such a system of numbers to our units of measurement immediately introduces a complication. The decimal fractions do not lend themselves readily to use with feet and inches in the way in which we now use, or think we use, those dimensions. However, it has been shown already that in many cases we are now dealing with decimal fractions in some of our most common articles of commerce, and there is no reason to suppose that we could not extend this practice if the results to be achieved warranted it. If we adopted the inch as a standard of length, for instance, and used decimal fractions of inches and multiples by tens we should have a system with many of the conveniences of the metric system.

After all, commerce is actually conducted as much in terms of nominal sizes as in terms of actual or approximate dimensions, so that no difficulty need be anticipated in that direction. Production at the present time is effected largely in terms of gages, and it is certainly just as easy to produce gages to check a dimension equal to, say, 1.585 as it is to construct a gage to check a dimension equal to 1.750, that is, $1^3/_4$ in. or 1 ft. 9 in. It would seem as though no great difficulties would be introduced into production or manufacture by the adoption of such decimal fractions. The only other function needing consideration is that of design. The designer in most cases works in terms of decimal fractions anyway, and with our present system is confronted with the necessity of converting his final results into the conventional fractions of an inch. Certainly he ought not to complain if a system of measurement expressed in decimal fractions is adopted.

Let us now return to the graphs of actual sizes of commercial products as given in Figs. 1 to 11, 18, and 19. Study of such graphs will show that those products which are largely used as components of manufactured or fabricated articles, such as bar steel, bolts, structural shapes, wire, etc., are generally made in such sizes that the graph to semi-logarithmic coördinates is concave toward the horizontal axis. This means that in the larger sizes such materials are made to finer sizes (smaller steps) in a geometrical sense than they are in the smaller sizes. On the other hand, in the case of finished articles such as household utensils, containers, machines of various sorts, etc., there is a greater tendency for the graph to flatten out or to become convex toward the horizontal axis. This indicates a tendency toward more uniform

spacing throughout the line and in some cases toward wider spacing, in a geometrical sense, in the larger sizes.

Such tendencies as these may not be lost sight of in any effort to rationalize sizes by the adoption of preferred numbers. It may be that we now produce an excessive number of sizes in certain products and that we would actually save in the long run by producing fewer, but such matters require long and detailed study before conclusions can be drawn.

There is a further thought which should be kept in mind during all efforts to adopt preferred numbers. Materials as used in production are basically dimensioned to meet certain loading requirements. It happens that physical dimensions appear in several different ways in the formulas which are involved, particularly with reference to their exponents.

As an illustration of the significance of this, strength in tension or compression varies with the square of the diameter in the case of a solid circular section, but strength with respect to bending varies as the cube of the diameter. If we imagine a set of preferred numbers in use and further imagine that round steel stock is rolled to preferred diameters, it hardly seems possible that one set of preferred diameters can give equally desirable variations of its squares and its cubes. When one considers the further complications which are introduced when other types of loading are brought into consideration, the case appears complex indeed.

Such difficulties may prove to be more apparent than real, but the underlying ideas should be borne in mind in any detailed study of this subject.

Possibilities in Simplification of Design

Attention thus far has been concentrated largely on size as a finished or completed proposition with little reference to the mechanism of design. No small part of the saving to be expected should accrue from simplification of design. This matter has been sparingly treated in the rather meager literature of this subject and a few very interesting studies have been recorded. In particular, a paper by Erich Hoffman published in *Mitteilungen des Normenausschuss der Deutschen Industrie*, February, 1920, contains two very interesting examples, one of these being a set of eyebolts.

The collar diameter of the smallest eyebolt is taken as the basic dimension, and for successively larger bolts this is increased in the ratio $\sqrt[10]{10}$ for the smaller range of sizes, and in the ratio $\sqrt[20]{10}$ for the larger range of sizes. Other dimensions of each eyebolt are obtained by taking its collar diameter as a starting point and multiplying by factors differing with the dimension sought. The factor used for a given dimension (such as internal diameter of eye), however, is the same for all sizes of bolt, and the result is therefore a set of geometrically similar eyebolts. As a matter of fact, the Germans have gone one step further, in that, for the set of eye-

bolts in question, each dimensional multiplying factor is taken as one term in the series with the ratio $\sqrt[80]{10}$, and thus each of these factors is itself a preferred number.

This sounds like a very pretty piece of mathematical juggling in connection with machine design, but it really has a very deep practical significance. A designer need design and draw only one size of eyebolt if he is sure that a certain series can be applied. All other desired sizes can then be obtained by the simplest form of calculation or directly from a table. If there is doubt as to the applicability of one series, he may design and draw the two extreme sizes and the middle size, determine the applicability of any chosen geometrical series, and proceed by simple interpolation to tabulate dimensions for all sizes. A certain amount of caution is necessary in such proceedings and it is always best to draw at least the two extremes if one is in doubt as to the applicability of a given series. Hoffman points out a case in which certain handwheels were under consideration. A satisfactory result was obtained by grading the outer diameter according to the $\sqrt[20]{10}$, the thickness of the rim and spokes according to the $\sqrt[40]{10}$, and the diameter of the hub according to the $(\sqrt[80]{10})^3$. This looks extremely complicated, but if one draws the largest and smallest handwheels, plots the points on semi-logarithmic paper and draws straight lines between, the job is finished.

German authors have been quite enthusiastic over the fact that geometrical series, and particularly geometrical series with ratios equal to roots of 10, have proved widely applicable to design and sizing as now carried out. They seem to emphasize the ease with which the series based on $\sqrt[80]{10}$ can be fitted into existing designs. At first sight it seems as though such applicability represented a most remarkable coincidence, but there is really much behind the phenomenon.

In the first place, human beings accept geometrical ratios in preference to arithmetical when the results are presented in such fashion that the mathematical construction is not in evidence. This is a phenomenon well known to psychologists and one which has been extensively tested. Examples could be cited from many different human activities, but consideration of size is sufficient for the present purpose. The normal human being, in developing a series of sizes, starts out with small increments, enlarging them as he increases the sizes. He thus unconsciously approximates some geometrical series or some combination of geometrical series.

In the second place, one can obtain practical approximations to all the numbers there are by using a sufficiently great number of series based on roots of ten.

The preference for the series with the $\sqrt[80]{10}$ ratio is probably due to the large number of terms in the series, and to the binary structure of the number 80, which contains 2 as a factor four times.

This last feature makes this series fit in with many existing sizes since, as has been pointed out, the binary principle (successive division or multiplication by 2) has been extensively used in fractional and in many other series of "preferred numbers" which have been developed and used more or less unconsciously through a long period of time. This feature also makes it possible, if changing industrial conditions make such a development desirable, to double the number of terms in a 5-, a 10-, a 20-, or a 40-series, or in any part of such a series. Furthermore, it may happen that this series represents approximately the path chosen by an uninfluenced human being and is therefore mathematically located in accordance with the theory of probability.

In presenting this paper the authors have attempted to picture the present status of the preferred-number idea and to do it in such a way as to point out its advantages and also its complications and dangers. The authors themselves hold no brief for preferred numbers, but they do believe that the idea indicates possibilities of simplification and elimination of waste of such magnitude that thorough investigation is justified.

DISCUSSION

BUCKNER SPEED.[1] Most mechanical operations are cuttings or dividings; on the other hand, the system of *counting* as distinguished from cutting originated in the piling up of things one by one, as for example, money, soldiers, goods.

For arithmetical operations, such as enumeration of population, or matters relating to money or goods, the decimal system is at every one's finger tips, deeply rooted in the very structure of our hands.

For mechanical operations involving cutting in its broadest sense, the use of the scale divisible by two is equally old as the sword arm of our fair-haired ancestors.

This is what is forcing itself on the users of the metric system, which is a beautiful doctrinaire product of the French Revolution and is useful in many cases, certainly in money counting, in certain kinds of buying and selling, and in some engineering and many other calculations.

Now this preferred-number system is an endeavor to adapt the dividing-by-two system to the metric system.

The chief purpose of a "size" is to enable the desirer of a certain size to most easily make his want exactly known to a supplier who has a number of "sizes" in his possession. The desirer would like to be able to find a supplier who had a stock graded by minutely small steps, but the supplier must in all reason only make and keep the fewest number of sizes that will retain his continued good-will with the desirer.

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[1] Charge Spec. Studies, Western Elec. Co., New York, N. Y.

This, and this only, seems to me is the right rule for the grading of things into sizes.

Not only is a rule such as the preferred-number system undesirable, but it often happens that neither a simple equal-increment rule as the number sizes of hats, nor a double-the-volume rule as in bottles, nor a three-sizes-up-and-halve-the-area rule of the Brown & Sharpe copper wire gage will do after the demand exceeds the supplier's wish to restrict the number of sizes kept in stock to any set rule.

There are also good size systems in which the magnitude of the intervals diminishes as the size of maximum demand is approached and the intervals become greater among the rarely-called-for sizes

These are matters of extreme practicality.

The argument of saving in cost of design misses the whole point, for after all the chief use of a number or a letter designating a size is to afford a memonic key by which a person may make known to another person his desire, and not solely to fall into some general system.

A. L. De Leeuw.[2] The troubles with past attempts at standardization have been twofold: either people did not do anything at all, or else they tried to go too far.

In the paper under discussion a thing is brought out which seems to be getting around the standardization problem quite nicely. The part of the paper which interested me was about the preferred system of figures, and particularly the word "preferred," which means that we *can* use it but do not have to. Whatever system of standards we may want to apply, we must take care that we let nature take its course, and many attempts at standardization have failed because the course of nature and its vagaries have been disregarded. As soon as we make a standardization system ironclad, a thing which has no outlet for human tastes, vagaries, and foolishnesses—as soon as we do this, the thing falls by its own weight.

That it is dangerous to lay down a system of standardization for future generations is vividly brought out by the present war between adherents and opponents of our English system of weights and measures. Of course we had to have a system, at least in this case, but once having it we are tied to it and cannot let go. I do not mean to say that the English system is so perfect that it could not be improved, but I do wish to say that we cannot let it go, because we have it.

Again I say I am not speaking of the advantages or disadvantages of any system of weights and measures. I am not claiming that the English system is better or worse than the metric. I am speaking of this fact: that, having the English system, we cannot get the metric, whether we want it or not. We are tied by our standards. We cannot very well get away from such a thing as a standard of

[2] Cons. Engr., 149 Broadway, New York, N. Y. Mem. A.S.M.E.

weights and measures, though this does not mean that we will have the same disadvantage when we standardize in other respects.

The system of preferred numbers has, at least in its name, this element of broadness: that we have a system of standardization but are not compelled to follow it.

Whether the proposed way of getting at the numbers is exactly right or not does not interest me very much, at least not at the present time. I believe, as the authors said, that it is merely an attempt to get the thoughts of the country, to get us to think about the matter, and to decide whether this is the best way or not, and if not, to get something better.

W. H. TIMBIE.[3] It would seem that the system of preferred numbers as presented in the paper could be bettered in one respect by a very simple change. It will be noted that in Table 4 there are several sizes numbered 1 and several numbered 2, etc. It seems to me that there would be a big advantage gained if in each system there were but one number 1, number 2, and so on. For instance, according to the scheme as presented a glass numbered 2 could be any one of four sizes. I would suggest as a numbering scheme that the thousandths of the logarithm be used as the number. Thus, in the first column of the table we should have as numbers 0, $12^1/_2$ 25, 50, and so on. The size number 200 in one column would be number 200 in any column, regardless of what series it was tabulated in. A system that has such a scientific method of sizing should certainly have a scientific method of numbering the sizes.

F. W. GURNEY.[4] The thought that strikes me quite forcibly is how purely academic this whole discussion is. Of course there is no denying the fact that our various standards of sizes are far from ideal. The same thing may with equal force be said of any of the customs or laws or fashions or habits of the human race. What a tremendous nuisance, for example, is the Babel of languages that we so haltingly speak and that we cannot speak. But how completely we realize the utter impossibility of bringing all our discordant tongues into one harmonious language. Yet this is not more futile than to think of changing over the sizes of our nails, our frying pans, our bolts, or our shafting. Ideally, it might be desirable. Practically, it is impossible. To put across such a change would entail an expense of billions, and the confusion that would result during the process of transition would be almost beyond estimate. And then it is very doubtful if these seemingly ideal standards when put into practice would prove to be as feasible as the present imperfect standards that have been worked out by the experience of generations.

The whole matter is very like the movement for the adoption of

[3] Prof. Elec. Engrg., Mass. Inst. of Technology, Cambridge, Mass. Mem. A.S.M.E.

[4] Ch. Engr., Gurney Ball Bearing Co., Jamestown, N. Y. Mem. A.S.M.E.

the metric system. It is susceptible of beautiful argumentation but is something which American industry won't stand for.

E. A. JOHNSTON.[5] In a general way I am fully in accord with the theory or principle involved, but feel sure that the application will be difficult. While it is too true that our present sizes, numbers, capacities, and gages are inconsistent and mean very little, I fear that it would be a stupendous undertaking to change these as applied to commodities already in common use.

GEORGE S. CASE.[6] The paper shows in a very interesting way how size standards automatically fall into geometrical progression. This would be even more convincing if the authors made the best of their case. Wire nails, for instance, have for their most important factor surface area, which is proportional to the product of the length and diameter. Fig. 1 looks as if this would work out very close to geometrical progression.

The case in Fig. 10 would be even stronger if the little-used dimensions were left out—$9/16$, $1 3/8$, $1 5/8$, $1 7/8$ and $2 3/4$-in. bolts have almost entirely dropped out of use and would not be stocked in one per cent of the volume of other sizes. If these sizes are eliminated, the curves in Fig. 10 become almost perfect straight lines.

It has been customary where sizes have been worked out scientifically to use geometrical progression and it is hard to see that anything has been gained by the change of nomenclature to "preferred numbers."

As an example of the possibility of standardization along this line, the company with which the writer is connected formerly used twenty-eight sizes of pasteboard cartons for packages of bolts. By setting up a new series of sizes, each one 15 per cent greater in capacity than the next one to it, it was found possible to cut in two the number of sizes and have the boxes more uniform than in the worst case under the old series.

F. R. STILL.[7] In connection with the subject of size standardization by preferred numbers, it may be of interest to know that the steam engines we make practically fit in with this scheme very nicely. Taking the horsepowers as follows:

$$10, \quad 16, \quad 25, \quad 40, \quad 64, \quad \text{and} \quad 100$$

our sizes of engines with corresponding horsepowers would be:

4 x 4, 5 x 5, 6 x 6, 7 x 7, 9 x 8, and 8 x 8 Double Cylinder.

CARL G. BARTH.[8] Being somewhat musically inclined I have looked upon the geometrical progression of an adjusted musical scale as a fundamental one handed us by nature herself. In this

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the simple number 2 is the basis as against 10 in the progressions presented in this paper as having been proposed for adoption on a large scale in Germany. In the musical scale the "interval" between two successive semitones is $\sqrt[12]{2} = 1.05947$, i.e., the ratio of their respective number of vibrations per second is 1.05947.

So long as we here in America stand together with England in refusing to adopt the metric system of weights and measures, I think we had better also adopt a system of geometrical series with 2 as the basic number rather than 10. Referring again to the musical scale, the vibrations of two successive octaves therein have this simple ratio 2.

Some ten years or so ago, in making a slide rule for the quick determination of the strength and deflection of coiled springs for one of my clients, I discovered, as pointed out in this paper, that the Brown & Sharpe wire gage is based on a geometrical progression. To my chagrin, however, I found that it seemed void of a simple ratio anywhere, though 2 came as close to being such a number that it seemed a shame that its constant ratio had not been made $\sqrt[6]{2} = 1.1264$, instead of 1, as I found it to be after getting in touch with Mr. Burlingame about the matter.

I will also mention that for more than 15 years I have tried to persuade, and mostly succeeded in persuading, my clients to adopt standards of hour rates for both machines and men that approximately follow geometrical progressions. Along with this effort I have also insisted upon eliminating any rates with fractional cents. A scale of hourly rate of this kind is therefore finally composed of a number of short arithmetical progressions pieced together so as to form a geometrical progression only in the averages of its arithmetical fields.

LUTHER D. BURLINGAME.[9] A discussion under the title of Preferred Numbers does not to my mind convey the principal and underlying thought which is most important to be brought out, namely, that a series in geometrical progression will for many purposes give the least number of sizes and at the same time meet the needs more fully than any other possible series of sizes. This has nothing necessarily to do with preferred numbers and this principle has been applied in practical ways for many years, although not as fully as might be desirable.

Such a principle was made the basis of the sizes of the American Wire Gage which was brought out by Brown & Sharpe Mfg. Co. in about 1857 and which provides for the diameter of wire following a geometrical progression, so that because of being in geometrical progression the steps between the small sizes are much less than between large sizes, as practical requirements dictate.

The application of this principle to the feeds of Brown & Sharpe milling machines was adopted by this company in the 80's, and

[9] Industr. Supt., Brown & Sharpe Mfg. Co., Providence, R. I., Mem. A.S.M.E.

in 1894 Mr. Carl Barth filed an application for a patent in which he emphasized the importance of a geometrical progression for geared feeds.

It is interesting to find how closely good practice has followed geometrical progressions in mechanical design, even though such designs were not originally laid out with that thought definitely in view. An example is in milling-machine proportions where the lengths of table traverse as agreed on by the manufacturers are in approximate geometrical progression from the smallest to the largest machine.

Another example of this is in the proportioning of taper shanks,
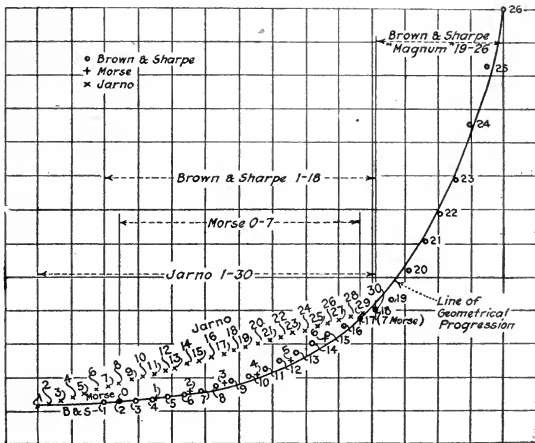
Fig. 21 Diagram Showing Variations of Diameters of Brown & Sharpe, Morse, and Jarno Tapers from a True Geometrical Progression of Sizes

regarding the standardization of which there has been much recent discussion. The accompanying diagram, Fig. 21, shows how closely the B. & S. tapers and Morse taper follow a geometrical progression.

The "Jarno" taper, on the other hand, varying by tenths of an inch from size to size, gives as large a step between the small sizes as between the large, and when carried up to 14 in. in diameter, as in the case of the B. & S. standard, would require about 140 sizes of the Jarno taper, an impractical series which would require a selection of sizes for actual use. The small unit of variation in diameter between the sizes of the Jarno taper, in this case 0.1

in., makes this illustration comparable with the need for preferred numbers for metric units where so small a unit as the millimeter is used as the basis of the system for mechanical work, thus expressing even moderate sizes in very large numbers. In such a case there is a great deal more need for a system of preferred numbers than in the case of our convenient system with the inch as the usual unit for mechanical work.

This objection to large numbers is pointed out by the authors, where they show the objection to using a measurement of 324 in., for example, instead of expressing the value in feet and yards.

At a recent meeting of the A.S.M.E. Committee on Plain Limit Gages for General Engineering Work the question of the steps where a change in limits and tolerances should be made in shafting was under consideration, and it seemed to be very natural to determine on a geometrical progression from $1/2$ in. to 8 in. with 2 as a multiple, thus making the sizes $1/2$, 1, 2, 4, and 8. In this case the metric equivalents would be approximately 13 mm., 25 mm., 51 mm., and 203 mm., so that rather than to use these odd numbers, selections could be made from a system of preferred numbers which would be approximations of the sizes desired.

If an attempt should be made to apply such numbers to the inch systems a range which would fit to this system would not be adapted to other needs—say, a system of machine keys.

Even with the metric system, where the need of preferred numbers is accumulative, there is difficulty in making the same series apply to varying conditions. Thus, much standardization along mechanical lines already adopted in Germany would be changed if brought in line with their proposed series of preferred numbers.

L. B. TUCKERMAN.[10] Standardizations always mean on the average a use of material in excess of actual need in order to save the excessive labor cost of fitting the material most sparingly to each particular use. With the recent increase of labor costs the range of economic standardization of practice has been widely extended, but whether this range has been sufficiently extended to include the sweeping standardization of a preferred-number system, is a very debatable question.

The standardization involved in a preferred-number system is so drastic and so complete that it should only be adopted after the fullest discussion of all the problems involved, including in particular a thorough study of all possible principal series, and all laws for the derivation of secondary and tertiary series. The need for such a thorough preliminary study of a proposed standardization is the greater the wider the scope of the proposed standards. It is also well known that it is easier to introduce some kind of standardization into a wholly chaotic practice than it is to introduce new standards into an already standardized practice.

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[10] Engineer Physicist, Bureau of Standards, Washington, D. C.

The more thorough the standardization the more difficult becomes any change or improvement, and a preferred-number system, embedded as it would be in all the standard practices of the nation's industry, would offer great obstacles to any change whatever. It is therefore of the greatest importance that a preferred-number system be adopted only when it has been shown that relative costs of material and labor justify its adoption, and even then only when it seems certain that the most advantageous series has been selected. A poorly chosen preferred-number system, although it might result in a considerable temporary advantage in furthering standardization where standardization is needed, might later become a detriment due to the great difficulty involved in replacing it by a superior system.

The thing which characterizes a preferred-number system over all other standardizations is its universality. It is intended to be applied to industrial products of the most diverse kinds, windows and automobile wheels, letter paper and tin cans, and especially to the innumerable products of automatic machine tools.

The immediate application to all of these is, of course, not intended, but the ultimate value of the system lies in the gradual absorption of all standardized articles into the one system of preferred numbers.

For this reason it is necessary that the preferred numbers be chosen on a basis as nearly universal as possible, and that it contain within itself all the necessary flexibility to adapt itself to the most widely differing applications.

A cursory examination of standard sizes of commercial materials such as had been made by Messrs. Hirshfeld and Berry immediately indicates the geometrical series as the only type of a series which over wide ranges of sizes will approximately represent the needs of a series of standardized products. The larger the number of examples collected, the more strongly is this conclusion borne out.

There can be no question, then, that a preferred-number system must be based on a geometrical series. The problem of the most suitable geometrical series to be chosen as the basis of the preferred-number system is not simple. The application of the system is so wide that very diverse considerations govern different applications, and the choice must finally represent a compromise in which the relative importance of many conflicting needs has been carefully weighed.

In a geometric series such as $a_n = a_0 r^n$ it seems obvious that there is no advantage to be gained in choosing any other number than 1 as the first term of the series. The series then would have the form $a_n = r^n$.

In the choice of the ratio, r, the series should contain only a finite and not too large number of different incommensurate ratios. r should therefore be of the form, $r = p^{1/q} = \sqrt[q]{p}$, where the

base of the system (p) and the exponent of the system (q) are relatively simple integers. Messrs. Hirshfeld and Berry discuss such values as

$$\sqrt[10]{10}, \sqrt[20]{10}, \sqrt[50]{10}, \sqrt[100]{10}, \text{etc.}$$

As our number system is a decimal system there is an obvious advantage in making the base of the system 10 (or perhaps 100), since then the digits of the preferred numbers repeat in each decade (or century) of higher order. So obvious is this advantage that practically all of the series so far proposed for a preferred-number system have 10 as a base. From certain other considerations it would seem that 12 might be a preferable base, but with the decimal-number system so firmly established that no change in it can be reasonably contemplated, all these advantages are outweighed by the obvious advantages of 10.

The full advantage of 10 as a base can of course only be secured by the use of a decimal system of units, and it is difficult to see how a preferred-number system can be made practically workable in the English system of units. It seems fairly clear that the adoption of a universally applicable preferred-number system as a basis for standardization presupposes for its successful working the adoption of a system of units with a common ratio of units such as the metric system, so that the base of the preferred-number system may be chosen the same as the standard ratio of units.

There is no such obvious indication for determining the proper exponent of the system as governs the choice of the base. Shall the primary series be a coarse series from which secondary and tertiary series are obtained by interpolating finer steps? Or shall the primary series be a fine series from which secondary and tertiary series are obtained by skipping steps in the primary series?

There seems to be no clear, unambiguous answer to either of these questions. The Germans have adopted 10 as the exponent of their primary series, i.e., $r_1 = \sqrt[10]{10}$ thus choosing a coarse primary series and obtaining their finer secondary and tertiary series by dichotomy,

$$r_2 = \sqrt[20]{10}, r_3 = \sqrt[40]{10}, r_4 = \sqrt[80]{10}, \text{etc.}$$

The only reasons given for this choice are that it seems to allow sufficient flexibility for the few illustrative examples examined and that it fits in well with a series of standard diameters (DINorm 3) already adopted by the Normenausschuss der Deutschen Industrie. These reasons are sufficient to establish a temporary advantage in the adoption of this system, but it by no means insures that the choice will not prove burdensome later as the field of application of preferred numbers widens.

There are very definite reasons for doubting whether this choice is a wise one and whether in particular the principle of dichotomy

is the proper principle to be used in interpolating a fine series into a coarse series of preferred numbers.[11]

It is seldom that a standard series of articles are geometrically similar because of the fact that it is seldom that they are constructed with only one limiting physical requirement. This is illustrated by the series of handwheels discussed in detail by Hofmann in his report to the Normanausschuss in which different dimensions are graded according to the different series $\sqrt[20]{10}$, $\sqrt[40]{10}$, and $\sqrt[80]{10^3}$. (Incidentally this last gradation also shows that they have already found the fundamental principle of their series insufficient for all cases, in that they have introduced a new coarse series $\sqrt[80]{10^3}$ by skipping terms in a fine series.) Illustrations of this fundamental fact can be multiplied indefinitely. Nails are varied in length and the diameter simultaneously varied (as noted by Hirshfeld and Berry) to provide adequate strength. Pipes are varied in diameter to provide for larger flow and at the same time are varied in thickness so as to stand the pressure.

It seems impossible to state any general law of double, triple, or multiple requirements on the dimensions of standardized articles, but in an overwhelming majority of cases the requirements are approximately expressed over a wide range by the relations $(k_1L_1)^{a_1} = (k_2L_2)^{a_2} = (k_3L_3)^{a_3} = (k_4L_4)^{a_4} =$ etc. Here L_1, L_2, L_3, etc., are different dimensions of the article, k_1, k_2, k_3, etc., are numerical constants, and the exponents a_1, a_2, a_3, etc., are in general simple numbers, 1, 2, 3, 4, 5, etc., rarely as large as 5. Thus in the example of Hofmann's handwheel $(k_1L_1)^4 = (k_2L_2)^2 = (k_3L_3)^3$ where L_1 is the outside diameter, L_2 the thickness of the rim or the spokes, and L_3 the height or diameter of the hub. Dr. Buckingham has calculated an interesting case of a ball bearing designed to support an overhung flywheel. Here he finds the relation $(k_1L_1)^4 = (k_2L_2)^5$ [where L_1 is the diameter of the balls and L_2 the diameter of the shaft] is necessary to insure proper stresses in the shaft.

These relationships suggest the desirability of making the primary series a fine series and making the exponent q divisible by as many small integers as possible. 60 suggests itself as a possible choice. Making $\sqrt[60]{10}$ the primary ratio, a number of coarse secondary and tertiary series can be found by taking every second, third, fourth, fifth, sixth, tenth, twelfth, fifteenth, twentieth, or thirtieth term in the original series. This would include the primary ($\sqrt[10]{10}$) and secondary ($\sqrt[20]{10}$) series of the present German proposal, and would avoid the awkwardness of such ratios as $\sqrt[80]{10^3}$.

It is not intended here to propose definitely that $\sqrt[60]{10}$ should be

made the primary ratio of the series, but merely to indicate some of the considerations which should be taken into account before a preferred-number system is adopted. It may very well be that there are advantages in a dichotomous derivation of secondary and tertiary series which would outweigh the presence of 3 as a factor in the exponent of the series, but the fact that 3 occurs so frequently as an exponent in physical laws certainly indicates that its advantages and disadvantages should be carefully weighed in selecting a preferred-number system.

It is the practice in Germany to round off the preferred numbers to the nearest unit or, where that gives too great a discrepancy, to the nearest tenth. In some discussions it is implied that this rounding off is merely a concession to a natural dislike of complicated figures. A closer examination shows that the demand for rounding off rests upon a much firmer basis. The dimensions of articles are conditioned not only by the requirements which each individually must meet, but also in large measure by their mutual relationships.

Fundamental in these relationships is the need of fitting. So fundamental is this need that in many lines of manufacture whole series of special fittings are manufactured to meet it. A simple illustration is the series of reducing L's, T's, and couplings used in fitting pipe of different sizes. Here the very nature of the objects demands specially adapted fittings. In many lines, however, proper dimensioning of standard sizes is all that is necessary. In the common brick, for instance, the breadth and length are so chosen that the header courses fit with the stretcher courses without the use of bats as fillers.

The basic requirement of fitting of different standard sizes without special adapters or laborious shaping at fitting points, is that the fitting dimensions shall have simple commensurate ratios. In the rounded series of preferred numbers this requirement is met as far as possible, without too wide departure from the basic geometrical series. The rounded values of the German primary series, $\sqrt[10]{10}$,—1, 1.2, 1.6, 2, 2.5, 3, 4, 6, 8, 10—have a maximum discrepancy of about 5 per cent from the geometric series at 3 and 6. Whether this discrepancy is too great a price to pay for their simple fitting possibilities can only be decided by careful investigation. It would be worth while to find out whether any different coarse series would permit of as simple rounding off without greater error.

In many fields of application this problem of fitting without special fittings does not of course enter. In any round stock, such as pipes, shafts, wheels, etc., special fittings are always in the nature of the case necessary.

This raises the question whether a single preferred-number series should be used, both for those cases where fitting is an important problem and for those cases where it is not required. It may very well be that not one but two or three or perhaps more rad-

ically different preferred-number systems might ultimately be more advantageous.

E. R. HEDRICK.[12] The reason for the selection of the relation $r = \sqrt[10]{10}$ is not always stated clearly. Any system of preferred numbers that did not include 10 as a ratio between two of the numbers would be open to serious question. But 2 is also a convenient multiple. It seems to me that the well-known fact that $2^{10} = 1024$ is the essential reason for the choice of the preceding ratio, since 10^3 ($= 1000$) is so very nearly equal to 2^{10}. Whence we have $10^3 = 2^{10}$ (approx.) or $\sqrt[10]{10} = \sqrt[3]{2}$ (approx.). For this reason the system based upon $r = \sqrt[10]{10}$ will also contain multiples of 2 to within the degree of accuracy commonly accepted in the process of rounding off. While it may be argued that the numbers 2, 4, do not occur among the preferred numbers, it has been pointed out repeatedly by many that the decimal point may be shifted in the preferred numbers as stated. After this is done it will be seen that the numbers 2, 4, 8, 16, 32, do occur, as also the numbers 0.5, 0.25, 0.125. If it were not for the rounding-off process the multiples of 2 would continue to appear. We cannot hope that they will continue forever in any rounded scheme, but it would be desirable to retain 64, which is under debate, if we can.

Finally, I may remark that any two geometrical progressions that both contain the same two numbers must also show other numbers in common. This remark becomes important in connection with the figures shown in the paper by Messrs. Hirshfeld and Berry. In that paper many of the figures are drawn on semilogarithmic paper. If we set $S = Ar^n$, where S is the size, r the fixed ratio, and n the size number, then $\log S = \log A + n \log r$. If we plot $\log S$ against n on ordinary paper or S and n against each other on semi-logarithmic paper, the figure will be a straight line and its slope will be $\log r$, i.e., $\frac{1}{10}$, if the scales are the same in the two directions and if logarithms of the base 10 are used with $r = \sqrt[10]{10}$. Since this would be a very inconvenient slope, it is preferable to take the units in the direction representing size numbers very much smaller. In this paper the units seem to be in about the ratio of one to ten, so that the line appears to have a slope of about one, that is to say, it makes an angle of about 45 deg.

The connection of this with my previous remarks is that the slope will be $\log r$ in any event, but the same straight line can be used for different values of r by changing the unit in the direction that represents size numbers. This would mean, for example, that the vertical lines in any of the figures may be omitted entirely and any desired number of new vertical lines can be inserted in their stead, provided only that the first and last lines be kept in place and that the new verticals be equally spaced. For the effect of doing this

[12] Professor of Mathematics, University of Missouri, Columbia, Mo. Mem. A.S.M.E.

is to change the scale in the direction that represents the size numbers. It seems to me that it is very important for those who use these figures to understand this that the slope of such graphs is not very significant. Otherwise they may attach too much significance to the apparent slope of these lines, and to the differences in the slopes in two different figures.

In connection with the application of preferred numbers to sizes of cartons, boxes, etc., if the controlling linear dimension would follow the preferred-number series, the volumes would follow a geometrical progression with the ratio $(\sqrt[10]{10})^3$ which is approximately equal to 2. The effect is the same as choosing for the volumes every third number of the preferred-number series, thus 2, 4, 8, 16, 32, 64 (according to the German series). As before noted, $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ also occur in the preferred-number list if the decimal point is moved in the customary manner. It should be noticed that intermediate sizes for such containers should be inserted *two at a time*, since doing so would again result in a geometrical progression whose ratio is $\sqrt[10]{10}$.

The size series for druggists' bottles is an interesting example. Thus the present sizes are 1, 2, 4, 8, *10, 12*, 16 oz. The only deviation from the regular preferred-number series being in the case of the 12-oz. bottle which would become 12.5 oz., since 10 and 12.5 are the preferred numbers between 8 and 16. From the present standpoint, these would count as intermediate sizes in the main series 1, 2, 4, 8, 16.

A. E. Kennelly.[13] The system of selecting sizes described in the paper would cause such widespread and radical changes in production that we should regard the system at present as a theory of economic design for future gradual introduction, rather than for immediate adoption. The system would be a much more radical innovation than the metric system of weights and measures. In order to introduce the metric system into production we should not have to change any size, but only the numerical values of the dimensions of existing things. On the other hand, to adopt a system of preferred new sizes in geometrical ratio for hats, shoes, frying pans, window panes, pipes, etc., would probably mean changing or rejecting machines and machine parts, in many cases at great expense.

Probably a geometrical series of preferred numbers would permit of realizing the minimum number of commercial sizes and of standardizing such sizes in the simplest and most scientific way. The 80th-root-of-10 series, with its constant ratio of $1.0292 = \log^{-1} 0.01250$, or a nearly 3 per cent step, seems to offer, as suggested in the paper, a very sound basis of advance; because a 3 per cent step is probably sufficiently small to provide for fine gradations where needed, and skipping intervals in the series systematically

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[13] Prof. of Elec. Engrg., Harvard University, Cambridge, Mass.

it gives successively by inclusion the 40th-root series of about 6 per cent ($1.0593 = \log^{-1} 0.0250$) the 20th-root series of about 12 per cent ($1.122 = \log^{-1} 0.0500$), the 10th-root series of about 26 per cent ($1.259 = \log^{-1} 0.100$) and the 5th-root series of about 60 per cent ($1.585 = \log^{-1} 0.200$). In all such cases the new series is most readily defined by the logarithm; thus the preceding sizes would be naturally designated as sizes 25, 50, 100 and 200.

From a purely theoretical point of view, it is a matter for discussion whether the basis of the geometrical ratio should be a root of 2 or a root of 10. In other words, some would prefer that the series of steps should pass exactly through the binary powers 2, 4, 8, 16, etc., whereas others would prefer that the series should pass exactly through the decimal powers 10, 100, 1000, etc. The recommendations of the paper are in favor of the decimal system. This is probably the better choice, from the standpoint of arithmetical computations and of engineering design, especially when it is seen that the decimal steps pass very nearly through the binary powers ($1.995 = \log^{-1} 0.300$, $3.981 = \log^{-1} 0.600$, $7.943 = \log^{-1} 0.900$, etc.).

G. M. EATON.[14] From the standpoint of adaptability of the preferred-number principle to the electrical manufacturing industry, we may divide the output into two major classes:

    1 Apparatus limited only fundamental natural law
    2 Apparatus subject to rigid physical limitations of an arbitrary nature, as well as by natural law.

The second class is well illustrated by axle-hung railway motors.

There is nothing fundamental about the track gage, but in view of the overwhelming existent investment in track, motive power, and rolling stock tied up to present standards, the gage ranks with fundamental natural law as a limiting feature.

If the railway industry were starting now untrammeled by precedent, some arbitrarily determined gage would necessarily be selected as the standard, and it certainly would not be our present standard. The motor ratings, car sizes, weights, etc., could in all probability follow the logarithmic law, though I am not yet sure whether this would be a wise arrangement. The ratio of active armature-core length to diameter could probably be fairly uniform for the smaller motors, being selected at a value that would give the best average weight and cost efficiency.

As soon, however, as we pass a rating where this ratio results in a motor which with its gear fills the available space between the wheels, we must depart from this ratio, the diameter increasing while the length gradually decreases as the gear face and other parts demand more room.

Graph 18, Fig. 22, shows the present W. E. & M. Co. standard

d.c. railway motor horsepower per 100 r.p.m., while graph 19 shows armature diameters, and 20, the lengths of active core.

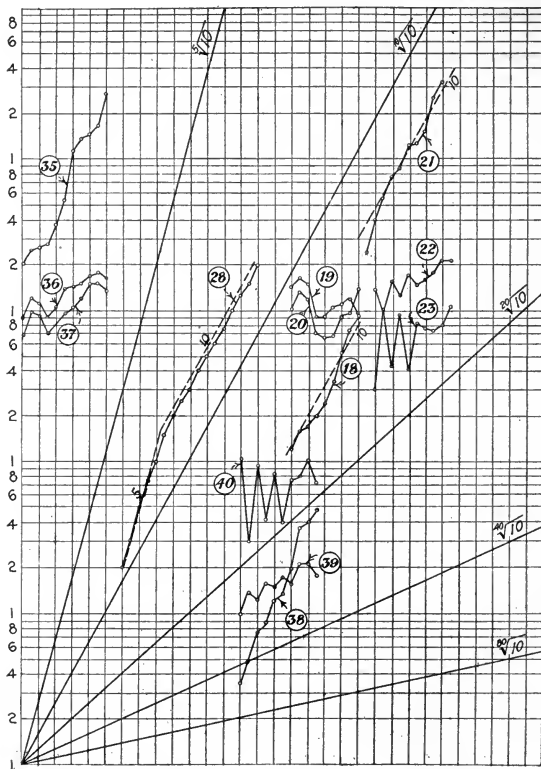Graphs 35, 36, and 37 show for the same motors the rating at

FIG. 22 GRAPHS OF HP. PER 100 R.P.M., ARMATURE DIAMETERS, AND LENGTHS OF ACTIVE CORE FOR WESTINGHOUSE ELEC. & MFG. CO. RAILWAY AND MINE-LOCOMOTIVE MOTORS

40 per cent of maximum r.p.m., and the corresponding core lengths and diameters.

The departures from the logarithmic characteristic form a very

long story of evolution, common in principle, no doubt, to many other lines of American products.

In addition to the gage limitation, there is of course a long list of other features, such as clearance dimensions, etc., which in effect are fundamentally binding.

A more fundamental limit is met in coal and other mining operations. I refer to thickness of vein, either height or width as the case may be, which produces a profound effect on the proportions of apparatus built to serve the mining industry.

Graphs 21, 22, and 23 (and 38, 39 and 40) show for W. E. & M. Co. mine-locomotive motors the features referred to for railway motors.

Railroad clearances and other shipping conditions also affect other lines of apparatus in the larger sizes, imposing restrictions which modify proportions. Below these limits, however, very logical lines of ratings have been developed, some of which approximate the logarithmic characteristic.

Almost all of the graphs show a drooping tendency. This is most sharply outstanding in cartridge fuses. All these lines have been developed to meet the demands of the trade. We must therefore satisfy ourselves as to the absence of sound reasons for a drooping tendency in the curves before we admit that a logarithmic characteristic would have produced better overall results or, indeed, results equally good.

Let us study the fundamental psychological and economic reason for the drooping tendency of the rating characteristic.

Let us assume that we have a customer who wishes to purchase a 12-hp. motor, and that we have in the 5th-root series only a 10-hp. and a 16-hp. rating available. With headway established for the preferred-number principle and with good salesmanship there should be comparatively little difficulty in satisfying him with the larger motor.

But when our next customer calls for a 120-hp. motor, it will be hard to show him that a 160-hp. motor is the right article. He will not be convinced by talk of ratios, because his mind will focus on the number of dollars extracted from his pocket to purchase a machine with a capacity in excess of his fundamental needs.

I hear the rebuttal say that the purchaser of ten 16-hp. motors will have to face a larger dollar differential than the man who buys one 160-hp. motor, and that the demand for small motors is numerically vastly greater than for large motors. The answer of course is obvious: viz., that in the final analysis we must study the reaction of the ultimate user, as the large orders for small machines are usually placed by manufacturers who assemble them in their own product, or by jobbers who resell them to the user.

Of course the rebuttal will say that in a preferred-number world no customer will have any use for a 120-hp. motor, and I will grant this when railway grades, the thickness of coal seams, the depth of

ore bodies, logical hydraulic head in power developments, and all the other things that combine to make the earth what it is, are arranged on a preferred-number basis.

In the case of the electric motor, a fundamental handicap is imposed on operating cost, as well as first cost, when an oversized motor is used. I refer to the bad power factor that is involved. This is a serious matter and would require most careful attention. It means that the application must be adapted to the motor. If this could always be done economically it would inevitably reduce the number of sizes, and ultimately save the people's money.

It is vastly more important in motor manufacture to reduce the variety of frame sizes, compound dies, coil formers, jigs, tools, etc., than to limit the ratings, and for many years the closest attention has been devoted to this end in bringing out new lines of motors. I realize that a motor in the 10-series would closely fit the requirements of the case of our customer referred to. This at first thought comes close to simply calling our present practice a preferred-number system and explaining away all departures from the true logarithmic characteristic.

But on closer analysis one cannot fail to be impressed with the remarkable coincidence of the characteristic curves of some important lines of apparatus with the fifth- and tenth-root series.

For example, graph 28 describing our type CS motors is very close to the fifth-root curve in the lower ranges, and is practically in exact coincidence with the tenth-root curve for the larger motors.

This feature is in evidence in some of the other lines. The point has been emphasized in graph 28 and a few of the other graphs by adding light dotted lines parallel to the fifth- and tenth-root curves.

We regard this as an important indication of the logic underlying the selection of these particular characteristics by the German Committee.

We have plotted a good many other lines of our standard apparatus and find such a prevalence of the drooping characteristic as to lead us to suspect that it represents a fundamental law, since the wide range of lines studied and the varying industries served by these lines should iron out individual tendencies. Later we find the tendency prevalent in the graphs presented in Messrs. Hirshfeld and Berry's paper.

Going now a little further into the details of our product, we will consider motor r.p.m. We at once face fundamental speeds in synchronous a.c. motors. These speeds, on any fixed frequency of supply circuit, are an inverse function of the number of poles, and the poles are in even numbers. The speeds therefore fundamentally lie in an arithmetical series, as the steps would be too wide if we limited the number of poles to a geometrical series of 2, 4, 8, 16, etc. In our practice, we change the progression of the series at 32 poles, stepping by twos up to 32 and by fours above that figure. It is interesting to note that the trade conditions which

demand closer size gradations in the larger sizes can be met in the same sizes by wider speed variations.

In industrial applications, trade conditions demand speed ratings for d.c. motors in line with the fundamental speeds referred to above. For example, a machine tool must be adaptable, without change of gear train, to either a.c. or d.c. power supply. Therefore, in electric-motor manufacture the vital factor of r.p.m. apparently cannot be influenced by any arbitrarily selected geometrical system.

Our rotating machines in particular, and in a lesser degree our stationary apparatus, are built up about fundamental relationships that are of the same binding nature as the r.p.m. discussed above.

E. H. RIGG.[15] The system of preferred numbers will be subjected to considerable investigation before being formally accepted by, for instance, the manufacturers of structural steel rolled to shape, such as channels, etc. Rolled steel is a subject which comes at once into the mind of a shipbuilder; it is one on which a vast amount of standardization work has already been done, and its total use in America is tremendous.

Taking American structural channel sections as an instance, we are accustomed to a series in which the depth of web in inches increases as follows: 3, 4, 5, 6, 7, 8, 9, 10, 12 and 15. The ten sizes constituting this series have been plotted in Fig. 23. It will be noted that the depths of web, weights, and section moduli do not run either in fair curves or straight lines.

The same thing has been done for British standard channels (narrow flanges) for the purpose of comparison and greater lack of fairness in the curves shows up (see Fig. 24). The ultimate requirement is strength, which is represented best by the section-modulus curve. It is a question worthy of further study whether a more economical set of channels could not be designed having section moduli varying somewhat as indicated by the dotted straight line joining the present minimum and maximum. The other features would vary approximately as the other dotted straight lines. It should be understood that these dotted lines have, for present approximate purposes, been drawn in by arbitrarily joining minima and maxima.

It will be noted that the straight lines applied to the American standards come considerably nearer to the mean of the curves than in the case of the British standards.

Varying thicknesses of each size will still be needed in order to get a finer grading of section modulus than would be obtainable otherwise. The possibilities are indicated by the dotted lines on Fig. 23. It will probably be found that varying the thickness of each full size will be more economical commercially than the introduction of half-sizes, but this point is worthy of further consideration. There has been some discussion looking to the adoption

[15] New York Shipbuilding Co., Camden, N. J.

of a single American standard line of channels for structural and ship work. An analysis along the lines of the preferred-number system would be in order and should by all means be made prior to the adoption of any economical single standard. In comparing
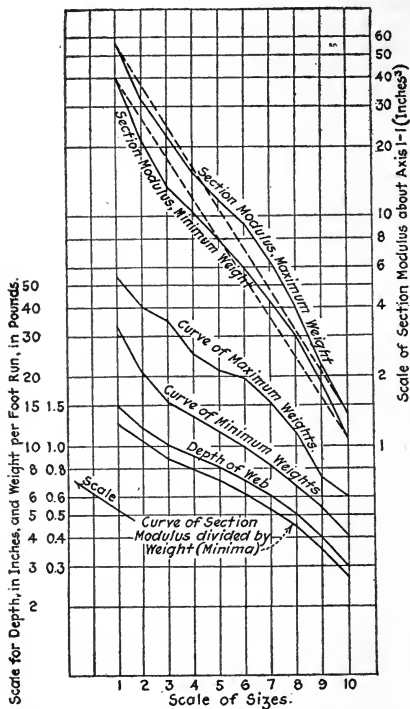

FIG. 23 AMERICAN STANDARD STRUCTURAL CHANNEL SECTIONS—CURVES FOR DEPTHS OF WEB, WEIGHTS, AND SECTION MODULI

the American standards (Fig. 23) with the British (Fig. 24), the greater influence of the shipbuilding demand in Great Britain must be kept in mind. Flange width and flange thickness in shipbuilding have to meet conditions different from those in buildings and bridges, and herein lie the difficulties of a single standard for America.

It is obvious that a full study of channel sections alone would be a decidedly lengthy affair, to say nothing of the other commercial rolled shapes. These remarks merely scratch the surface.

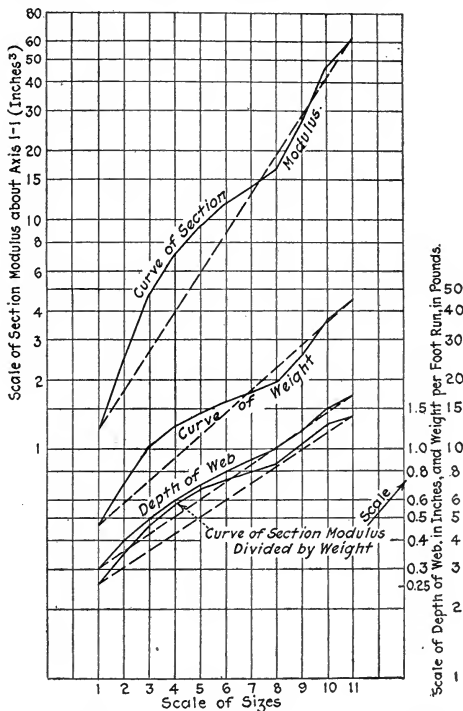WILLIAM A. DEL MAR.[16] The system of preferred numbers re-

FIG. 24 BRITISH STANDARD CHANNEL SECTIONS—CURVES FOR DEPTHS
OF WEB, WEIGHTS, AND SECTION MODULI

cently introduced in Germany and France presents no novel features to the wire and cable industry of the United States, as such a system has been in use in that industry since 1857 when the Brown & Sharpe gage was introduced.

The preferred numbers of the Germans are a series of numbers in

[16] Habirshaw Electric Cable Co., Yonkers, N. Y.

geometrical progression, i.e., related to one another by a constant ratio having the values $\sqrt[5]{10}$, $\sqrt[10]{10}$, or $\sqrt[20]{10}$, depending upon the series. The Brown & Sharpe gage, now known as the American Wire Gage, has the constant ratio $\sqrt[39]{92}$ between the diameters of successive sizes. This equals 1.1229 and $\sqrt[20]{10} = 1.1221$, so that it is obvious at once that the two systems are substantially the same, the difference being less than 0.08 per cent.

A comparison between the diameters of A.W.G. sizes and the secondary series of preferred numbers is shown in Table 6. As would be inferred from the similarity of the ratios noted above, the preferred numbers come very close to the A.W.G. diameters expressed in mils to the same number of significant figures. Thirteen out of thirty-four sizes show the two systems in exact agreement, and the maximum divergence is 2½ per cent.

When it comes to the application of preferred numbers to stranded

TABLE 6   COMPARISON OF A.W.G. SIZES WITH FRENCH AND GERMAN PREFERRED NUMBERS

A.W.G. No.	Diam., mils.	Preferred Numbers German	French	A.W.G. No.	Diam., mils.	Preferred Numbers German	French
30	10	10	10	13	72	72	72
29	11	11.2	11.2	12	81	80	80
28	13	12.5	12.5	11	91	90	90
27	14	14	14	10	102	100	100
26	16	16	16	9	114	112	112
25	18	18	18	8	128	125	125
24	20	20	20	7	144	140	140
23	23	22.5	22.4	6	162	160	160
22	25	25	25	5	182	180	180
21	28	28	28	4	204	200	200
20	32	32	32	3	229	225	224
19	36	36	36	2	258	250	250
18	40	40	40	1	289	280	280
17	45	45	45	0	325	320	320
16	51	50	50	00	365	360	360
15	57	56	56	000	410	400	400
14	64	64	63	0000	460	450	450

copper conductors, certain difficulties stand in the way of using the system in any way except for the diameters of the component wires, because the number of wires in a concentric cable must be according to the series 7, 19, 37, 61, 91, 127, etc.

G. F. Jenks.[17]   Many years ago an ordnance engineer proposed a series of guns advancing by geometrical series according to weights of projectiles or cubes of calibers. Although various countries have not standardized their cannon in exactly the same manner, yet the same underlying principle of calibers advancing in a geometrical series exists in all countries. This theory has withstood the most severe test that can be given any industrial product—war.

The design engineer is wasteful in production effort. He calculates carefully the strength of parts of his design, and dimensions

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[17] Maj. Ord. Dept., U. S. A., Washington, D. C. Mem. A.S.M.E.

it to withstand *exactly* the stresses introduced. And to cover the inexactness of his formulas of computation, he introduces a factor of safety. There is no good reason why this factor cannot be modified slightly to permit of the use of materials made according to definite lines of preferred numbers. One of our most careful engineers, when asked for his opinion on this paper, said of course it was good for other design work but could not be applied in such a careful designing as is required in ordnance construction. The recuperator, for example, requires special sizes for the piston. Upon its area depend the cylinder diameter and the thickness of walls of the cylinder. High pressures and comparatively low factors of safety are used. But when this problem was carefully analyzed it was found that a variation of $\frac{1}{2}$ in. in 6 in. made no practical difference in the design.

In some recent standardization work the necessity for standard $1^3/_8$- and $2^3/_4$-in. bolts was discussed. The percentage variation in area between $^5/_8$- and $^3/_4$-in. bolts is the same as between $1^1/_4$- and $1^1/_2$-in. bolts or between $2^1/_2$- and 3-in. bolts. If $^{11}/_{16}$-in. are not needed, why are $1^3/_8$ and $2^3/_4$? The designer says that he must have $^1/_4$-in. steps in large sizes and the producer says that he manufactures large quantities of $1^3/_8$-in. bolts. The area of the $1^1/_2$-in. bolt is 44 per cent greater than that of the $1^1/_4$-in. bolt. It is doubtful if many designers could justify the use of the $1^3/_8$-in. size or of the $^1/_4$-in. step in sizes above $2^1/_2$ in.

CARL J. OXFORD.[18] At present we have many articles which do not show a uniform rate of variation in size and for no apparent reason.

As a practical example of a uniform system of standardization applied to one particular product, I might here call attention to a paper on the Standardization of Small Tools recently read at a regional meeting of the Society. Without being acquainted at that time with the system of preferred numbers, I proposed as part of a standardization program the elimination of many sizes of twist drills now in more or less common use. Without regard to lengths or other differences of design, it was found that from $^1/_2$ in. down there are 137 sizes or diameters of drills regarded as standard articles. Some of these are practically identical in size but are designated by different symbols, while others show a non-uniform rate of variation. A tentative standard was then proposed. This standard was based on a system of groups of sizes, each group varying in a straight geometrical progression from the ones preceding and following, while the variation of sizes within a group was uniform. By this process 64 sizes were eliminated and 73 sizes were retained.

It is interesting to note the effect of applying the preferred-number system to this same range of sizes, and Table 7 has accord-

ingly been worked out using the ratio $\sqrt[40]{10}$ or 1.059. The nearest corresponding commercial size used at present has been inserted in each case, and it is noted that with exception of two sizes the existing diameters are very close to the theoretical diameters. This would result in the retention of only 64 sizes, which still would be ample for all practical purposes, provided the designers and mechanics could be educated to their use.

The proposal of basing the preferred-number system on some root of 10 is a natural one when it is considered that it originated in the European countries using the metric system. It does not follow, however, that this system of ratios is best suited for the English system of measures. To agree with the latter system it might be better to base the ratios on some root of 12 as $\sqrt[12]{12}$, $\sqrt[24]{12}$, etc. $\sqrt[12]{12}$ would give 12 proportionate sizes between one foot and one inch, and retain these basic units as an integral part of the system.

In closing, it might be said that any system is better than no system at all, which is precisely the condition now existing in variations of sizes of many manufactured articles.

TABLE 7   STRAIGHT-SHANK DRILL SIZES ARRANGED BY PREFERRED NUMBERS

Actual size	Nearest size now in use	Actual size	Nearest size now in use	Actual size	Nearest size now in use	Actual size	Nearest size now in use
0.502	1/2	0.200	8	0.080	46	0.032	67
0.474	none	0.189	12	0.075	48	0.030	69
0.448	29/64	0.179	15	0.071	50	0.028	70
0.423	27/64	0.169	18	0.067	51	0.026	71
0.399	X	0.159	21	0.063	1/16	0.024	73
0.377	Y	0.150	25	0.060	53	0.023	6 mm.
0.356	T	0.142	9/64	0.057	none	0.022	74
0.336	R	0.134	29	0.054	54	0.021	75
0.317	O	0.127	30	0.051	55	0.020	76
0.300	N	0.119	31	0.048	3/64	0.019	5 mm.
0.283	9/32	0.113	33	0.045	56	0.018	77
0.267	17/64	0.107	36	0.042	58	0.017	78
0.252	1/4	0.101	38	0.040	60	0.016	none
0.238	B	0.095	41	0.038	62	0.015	1/64
0.225	1	0.090	43	0.036	64	0.014	79'
0.212	3	0.085	44	0.034	65	0.013	80

REGINALD TRAUTSCHOLD.[19]   In the German system of preferred numbers, in order to make the various terms constituting a series convenient to use, the expedient is resorted to of rounding them off. That is, the individual terms are simply approximations, the nearest convenient decimal fractions being selected. This seems to be an inherent weakness, necessitating an arbitrary standard for "rounding" the terms.

A series of numbers in geometrical regressions is as effective as one in geometrical progression—in reality such a series is infinitely more effective, as it is all-embracing and can be carried on indefinitely—and the "head" of such a system should be 1, not 10. A geometrical regression with 1 as a head can be carried out to several terms without evolving an unfamiliar or inconvenient fractional

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[19] 69 Edgemont Road, Montclair, N. J.

term. For example, taking 1 in. as the head, the regressive series will be $1/2$, $1/4$, $1/8$, $1/16$, $1/32$, $1/64$, $1/128$ in., etc., all familiar fractions in English measurements. If 1 ft. or 12 in. is taken as the head, the regressive series is 6, 3, $1^1/2$, $3/4$ in., etc. These are all "preferred numbers" in the sense of the suggested German system, and it will be noted that the inch and foot series dovetail into one another. There is a common ratio between consecutive numbers, or terms, and in addition every second term in the series has a definite relation, also every third, every fourth, etc., so that the series includes "preferred numbers" for roots and powers as well.

Other systems of English weights and measures show in general a similar regard for preferred-number units, so it would appear that the English systems are basically preferred-number arrangements of extreme flexibility. It may be true that our use of the English measures is not as effective as it might be. We have confused their economic worth by the introduction of arithmetical simplification in calculations, but this does not alter the soundness of their geometrical derivation.

In the metric systems of measures arithmetical progression only is employed, and this is their chief weakness. The interest in preferred numbers evidenced by the Germans and other nations employing a metric system of measures confirms this statement, for it is an effort to introduce a system of accepted measures which will conform to the requirements for geometrical progression that has brought about the preferred-number movement. We already have the most flexible system of preferred numbers. Why adopt now a system with inherent weaknesses unless we propose to supplant the English system of measures by a metric system?

In our efforts toward standardization it would seem a pity to lean now toward a system which, recognizing its limitations, is endeavoring to adopt certain of the advantages inherent to the English system.

It is quite beyond me to see how the adoption of a system of preferred numbers derived in a manner similar to that adopted by the Germans could more effectively, or even as effectively, bring these improvements about than a suitable selection of preferred numbers from the flexible, comprehensive, and familiar geometrical series upon which the English system of measures is based. A standard selection of preferred numbers of English origin is to be strongly advocated, but this is a relatively simple matter compared to the complication of our present measures by the further adoption of units of metric derivation which would take years to introduce and entail a cost of billions.

OSCAR B. BJORGE.[20] An investigation has been conducted to see what may constitute an ideal series of hoisting engines from the smallest cylinders, 3 in. by 4 in., up to the largest cylinders, 14 in.

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[20] Gen. Mgr., Willamette-Clyde Co., Portland, Ore. Mem. A.S.M.E.

by 16 in., choosing the intermediate sizes by the system of preferred numbers. The principal factors of hoisting engines are the cylinders, crankshaft pinions, drum gears, and drum barrels, and when these are determined the rope pull of the engine is fixed. After choosing a series of cylinder sizes I have assumed a size of pinion, gear, and drum for the smallest and the largest, and chosen the intermediate sizes by means of preferred numbers. As a result of this a line of cylinders has been picked without fractional bores or strokes, the ratio of bore to stroke being a uniformly changing one. The resulting hoists exhibit a fine progressing uniformity, and, strange as it may seem, the resulting load capacities are more suited to the requirements of hoisting engines, determined by the weights and capacities of buckets, elevator cages, and other weights to be hoisted, than in the existing lines of hoists. There is probably no reason for this last coincidence, and yet might it not be true that if a series be correctly laid out, thereby obtaining uniformity, that the load capacities would be more suited to the varying loads for which hoists are used?

ARTHUR BESSEY SMITH.[21] The automatic equipment of a telephone central office which we make is practically always custom-made as to size and arrangement. It seems not to be possible to build central offices in graded sizes, because each customer trims his requirements to the least possible size, so as to save initial expense. However, each office is made with a definite ultimate capacity in view, and it is possible that this might be worked out on a definite series and the customer induced to accept an ultimate which is in the series.

The private automatic exchange has not yet been standardized, but may possibly be eventually. That such apparatus can be put on a geometrical series seems doubtful, because of the decimal nature of the switches of which it is made and the few sizes involved.

We doubt the applicability of preferred numbers directly to interior telephone cables, for their sizes are rather closely related to the number of terminals in racks and boards.

Storage batteries, charging machines, and wires and cables used to carry power current are all bought from other manufacturers. We must accept their sizes and select those which most closely fit our ultimate demand. Preferred numbers may be of real assistance here.

HARRY M. ROESER.[22] Rolling mills have been producing standard structural shapes for a number of years, and as there has been no prominent movement for restandardizing them it appears fair to assume that the shapes now rolled are those that human experience has hit upon as being most efficient, particularly in regard to

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[21] Chief Research Engineer, Automatic Electric Co., Chicago, Ill.
[22] Assoc. Engr. Physicist, Bur. of Standards, Washington, D. C.   Assoc-Mem. A.S.M.E.

the economy of metal used to meet trade requirements for strength and minimum costs. This being the case, if the preferred-number system is to be used for dimensioning these shapes, in order to justify itself it must yield at least as great efficiency in the distribution of metal as the present system.

The writer's opportunities confined the investigation only to structural channels which are now rolled in ten depths, viz., 3, 4, 5, 6, 7, 8, 9, 10, 12, and 15 in. Flange widths and web thicknesses are graded in a linear relation with respect to the depth approximately as follows:

$$\text{Flange width} = 0.90 + (0.17 \times \text{depth})$$
$$\text{Web thickness} = 0.14 + (0.01 \times \text{depth})$$

The above dimensions are for what are called "Minimum Standard" Channels. It is the trade practice to roll for each depth a number of different sizes of web thickness and flange widths.

After some preliminary calculation it was concluded that to reproduce the present series of channel depths by consecutive terms a series of the form $10^{i/x}$ was impracticable and consequently the terms of the $10^{i/80}$ series that best fitted the depths were selected. These terms are the 38th, 48th, 56th, 62nd, 68th, 72nd, 76th, 80th, 86th, and 94th. This practice may be considered not exactly consistent with the preferred-number system, which apparently demands that consecutive sizes be graded according to consecutive terms of a geometrical series. However, it is contended that the sizes are graded according to the terms of the 80-series. It merely happens that the sizes corresponding to some of the terms of the 80-series have no practical demand and are not manufactured.

The procedure of determining the series to fit the flange widths was as follows:

The series was assumed to be of the form $10^{i/x}$, where x is a number to be determined and the value of i which gives a certain flange width must be the same value of i that gives the corresponding web depth in the 80-series. That is to say, if the 38th term of the 80-series gives a certain web depth, then the 38th term of the x-series must give the corresponding flange width. On this basis the value of x was computed by the method of least squares to fit most closely the existing web thicknesses. The value computed for x was 191. The value used in this investigation was 192.

The method was repeated to determine the web thicknesses. The value of x for this series was 180.

Properties of the preferred-number sections compared to similar sections in current use are set forth in Table 8 and show that there is an economical advantage in favor of the current system of shapes.

ALBERT W. WHITNEY.[23] While the American Engineering Stand-

ards Committee does not itself deal with the technical details of standardization work, it has been very glad to coöperate with The American Society of Mechanical Engineers in arranging for a discussion of this important paper from the point of view of a number of industries.

That there can be an optimum selection among the multiplicity of possible sizes of manufactured products that shall be upon such a fundamental basis as to be applicable over a wide range of products is one of the most important and hopeful facts in the field of standardization. Such a judgment with regard to the possibility and importance of such a system of preferred numbers is not based solely upon a priori reasoning, for in Europe, where the principal developments in this field have taken place, actual results are showing both the importance and the feasibility of making use of such a selection: In France and in Germany, for instance, the development and application of preferred numbers is regarded as one of the most important achievements in the field of standardization.

The choice of a number series is a matter that calls for something more than empirical treatment. The fact that for a wide variety of purposes the best series of sizes is geometrical is most interesting and significant. One cannot help feeling that this is only another manifestation of those facts of human nature which are expressed by the Fechner-Weber law. This law, it will be remembered, states that the increment of stimulus that produces an increment of sensation is proportional to the amount of the existing stimulus; for instance, if when I am holding a pound weight I can just notice the addition of an ounce, then when I hold a two-pound weight I shall be just able to notice the addition of two ounces.

The fact is that in the type of preferred-number series that is under consideration we have apparently got hold of something that is fundamental: it has far more sanction than mere empiricism.

It has, for instance, an earmark that is found in every fundamental solution, namely, that it clears up a larger field than we had hoped initially to affect. As illustrating this I will instance the facts: first, that the number series that obtains where increased refinement is called for can easily be made to include the numbers in all lower series; second, that if linear dimensions are expressible by a geometrical series, then areas and volumes are also expressible by number series which are contained in the original series; and, third, by the apparent adaptability, due to fundamental considerations, of such number series to the simplification of machine design.

One other important fact should be noted. The fundamental character of this method would adapt it for use as a basis for international standardization. In fact, it has been suggested by two foreign national standardizing bodies that no time should be lost in getting the countries of the world together, not only in agreement upon a particular number series to be used, but upon a standardization of roundings. If such a common number series were

TABLE 8 PROPERTIES OF "MINIMUM STANDARD" CHANNELS

[Comparison of current shapes with those dimensioned by preferred numbers (P.N.)]

i	Depth d — Actual	Depth — $10^i/80$	Flange Width b — Actual	Flange — $10^i/192$	Web Thickness t — Actual	Thickness — $10^i/180$	h — Actual	h — P.N.	Area — Actual	Area — P.N.	I — Actual	I — P.N.	S — Actual	S — P.N.	S/A — Actual	S/A — P.N.	Per cent difference
38	3.00	2.99	1.41	1.58	0.17	0.16	2.66	2.67	1.19	1.268	1.6	1.305	1.1	0.873	0.92	0.69	+25
48	4.00	3.98	1.58	1.78	0.18	0.18	3.64	3.62	1.55	1.719	3.8	4.318	1.9	2.17	1.23	1.26	−2.4
56	5.00	5.01	1.75	1.96	0.19	0.20	4.64	4.61	1.95	2.222	7.4	8.633	3.0	3.45	1.54	1.55	−0.6
62	6.00	5.96	1.92	2.10	0.20	0.22	5.60	5.52	2.38	2.727	13.0	14.75	4.3	4.95	1.81	1.82	−0.6
68	7.00	7.08	2.09	2.26	0.21	0.25	6.58	6.60	2.85	3.349	21.1	25.53	6.0	7.21	2.11	2.15	−1.9
72	8.00	7.94	2.26	2.37	0.22	0.26	7.56	7.44	3.35	3.794	32.3	35.45	8.1	8.93	2.42	2.35	+2.9
76	9.00	8.91	2.43	2.49	0.23	0.28	8.54	8.39	3.89	4.305	47.3	51.05	10.5	11.46	2.70	2.66	+1.5
80	10.00	10.00	2.60	2.61	0.24	0.28	9.52	9.44	4.46	5.010	66.9	72.66	13.4	14.53	3.01	2.90	+3.7
86	12.00	11.88	2.94	2.80	0.28	0.30	11.44	11.28	6.03	6.106	128.1	124.64	21.4	20.98	3.55	3.44	+3.1
94	15.00	14.96	3.40	3.09	0.40	0.33	14.20	14.30	9.90	8.028	312.6	252.34	41.7	33.74	4.23	4.20	+0.7

$h = d - 2t$; $l = h - \frac{1}{2}(b - t)$; A = area of section $= dt + \frac{(b - t)}{6}(\ldots)$; I = moment of inertia $= \frac{bd^3}{12} - \ldots$; S = section modulus $= \frac{2I}{d_i}$.

S/A = metal-efficiency factor. Plus signs preceding values in last column of table indicate that present shapes are more efficient.

in general use, international standardization along other lines would seem likely to follow.

All these facts taken together seem to indicate that we have before us in the abstract a singularly fundamental and powerful instrument, one that seems likely to exert a profound effect upon standardization.

How far and how rapidly, however, we can go in the actual application of any such plan is a matter for very careful consideration. At the best it could go into effect to any considerable extent only on new lines and only very gradually on old lines as other occasions arose for making changes. Its chief value would lie in the fact that it was a norm to be gradually approached.

A. RATEAU.[24] It will interest the authors, no doubt, to learn that the original idea of preferred numbers emanated more than 40 years ago from Colonel Charles Renard, celebrated throughout the world for his numerous works, and particularly for his dirigible balloons.

It is right to render to each person his due and I should like to see the foregoing statement printed in a subsequent issue of *Mechanical Engineering*, as it would certainly be of interest to American engineers in general.

C. F. HIRSHFELD.[25] I shall not attempt to reply to each

[24] Pres., Societé Rateau, Paris, France. Hon. Mem. A.S.M.E.

[25] Ch. Research Dept., Detroit Edison Co., Detroit, Mich. Mem. A.S.M.E.

individual who has discussed the paper, but shall content myself with replying to the discussion in a very general way. First, standardization is arbitrary. That is the basis on which it starts. The question is this: If we give due consideration to the proper claims of all interested parties, is this arbitrary standardization worth while or not worth while? So far as we know, judging from human experience, I believe the majority of people think that it is worth while.

If it is worth while, and it must be arbitrary, the question is really, What does such arbitrary standardization lead to in the relations between supplier and user?

I will grant that the commercial tendency is to supply all the demands of the users and, as one would expect, a consequent large demand.

I happen to represent two organizations which last year undertook to standardize the steam turbine. We started by trying to standardize sizes of the steam turbine. We had the makers with us, and the makers were as usual between the devil and the deep sea. The users demanded certain very closely spaced sizes, and the manufacturers naturally wanted to meet the users' demands. On the other hand, the users were complaining about the cost of the unit.

It is perfectly obvious that if a man makes six sizes over a given range instead of twelve, the units will be cheaper in the case of such things as steam turbines; and we finally agreed upon certain standards, and sooner or later, if I am not mistaken, those sizes will be the standards of steam turbines in this country.

It does not mean that you cannot buy other sizes, but it means that when you want a steam turbine you will buy the standard or you will pay the burden that you put on the industry.

I think that this is the keynote in the adjustment between user and supplier. Give the user what he wants if he is willing to pay for it, and you will soon find that in most cases he does not want it.

There are some figures in the paper, notably Fig. 8, which show the results of an attempt to meet the needs, or supposed needs, of everybody.

I do not know whether they are justifiable needs or not; I do not pretend to know, but I do know that the sizes there shown are so closely spaced that there is at least the inference that they are not all necessary.
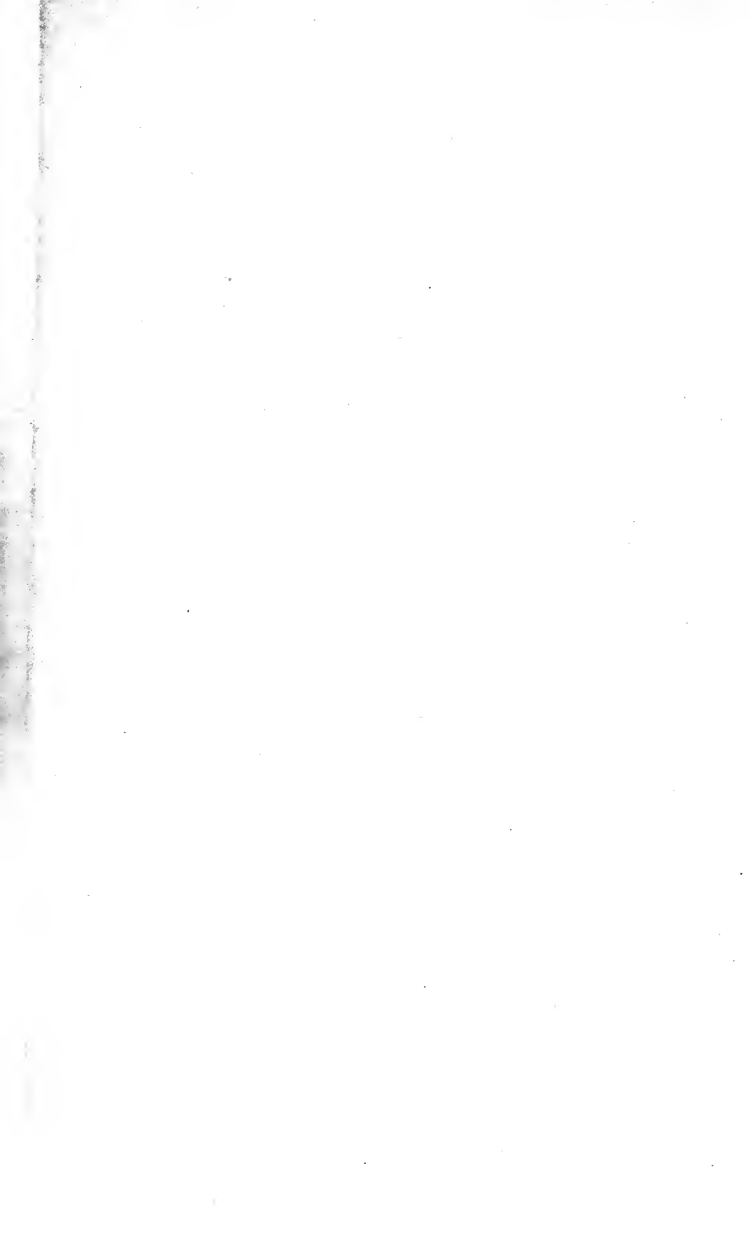
Considerable discussion has been given to the question of what series we should use. I think that is of very little importance right now. The question is, Do we want to use a series? Do we want to use preferred numbers? And if we do, should we arrange them in a geometrical series? If we do not use such a series, what one do we want to use for the preferred-number series? That is entirely a separate question from the matter of units.

I am not prepared to say that the 80th root of 10 is the ruling one,

or the 12th root of 12 is the best. I do not know enough about it. But if you first decide that there is enough in preferred numbers to make an effort to secure their adoption, the rest of the problem will take care of itself. With a few men like Mr. Barth around, we will get the system that fits.

I understood Mr. Speed to say that he was totally opposed to every standardization. I do not know whether I correctly quote him or not, but if that does express his attitude, I think possibly it comes from the fact that in the industry in which he works everything would lean toward that sort of a conclusion.

However, with this I am satisfied: It is not a question of whether some of us want standardization and some of us do not. Standardization is forced on us. If we are going to survive commercially we must standardize, and if we must standardize, let us get off and view the problem from the greatest possible distance. Consider it sanely and cold-bloodedly and decide we are going to standardize this way or that way, and after we have done that, having decided which way we are going to standardize—not whether we will or not, because we have got to—then let us discuss the details of that standardization.